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Evolution of primordial magnetic fields

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Here we briefly summarise the main phases which determine the dynamical evolution of primordial magnetic fields in the early universe. On the one hand, strong fields undergo damping due to excitations of plasma fluctuations, and, on the other hand, weak magnetic fields will be strongly amplified by the small-scale dynamo in a turbulent environment. We find that, under reasonable assumptions concerning the efficiency of a putative magnetogenesis era during cosmic phase transitions, surprisingly strong magnetic fields 10^{-13}–10^{-11} G, on comparatively small scales 100 pc–10 kpc may survive to prior to structure formation. Additionally, any weak magnetic field will be exponentially amplified during the collapse of the first minihalos until they reach equipartition with the turbulent kinetic energy. Hence, we argue that it seems possible for cluster magnetic fields to be entirely of primordial origin.

1 Introduction

Magnetic fields are ubiquitous in the local Universe (Beck et al. 1996) and there is growing evidence of their presence also at high redshifts (Bernet et al. 2008; Murphy 2009; Robishaw et al. 2008). The origin of those cosmic magnetic fields is still an open issue. Although there are various possibilities to generate magnetic fields by astrophysical processes (Biermann battery, Weibel instability) during and after structure formation (e.g. Bertone et al. 2006; Schlickeiser & Shukla 2003) it is still viable that those fields are of primordial origin or at least are generated during very early epochs. In particular, recent observations of Faraday rotation measure (FRM) due to high redshift galaxies show that strong magnetic fields are already present at redshifts $z > 1$ (Bernet et al. 2008; Kronberg et al. 2008). Also γ-ray observations from the FERMI satellite indicate that weak magnetic fields are present in voids of galaxies (Neronov & Vovk 2010; Tavecchio et al. 2010; Taylor et al. 2011), again suggestive of a very early origin of cosmic fields (but see also Broderick et al. 2012).

There are many suggestions to generate magnetic fields by non-astrophysical processes in the Early Universe. Weak seed fields could be produced during inflation with coherence lengths exceeding the horizon (e.g. Gasperini et al. 1995; Turner & Widrow 1988) and by causal processes during cosmic phase transitions (e.g. Sigl et al. 1997, see also the reviews by Grasso & Rubinstein 2001 and Giovannini 2004). If the observed magnetic fields are indeed of primordial origin their evolution during the cosmic epochs is far from trivial. For instance, on the one hand, saturated and strong fields will be dynamically damped by turbulent decay (e.g. Banerjee & Jedamzik 2004; Jedamzik et al. 1998) and, on the other hand, weak fields will be amplified by the small-scale dynamo (e.g. Brandenburg et al. 1995; Federrath et al. 2011a; Matsuda et al. 1971; Schleicher et al. 2010; Sur et al. 2010, 2012).

2 Damping of magnetic fields

In Banerjee & Jedamzik (2004) and Banerjee & Jedamzik (2003) the non-linear evolution of cosmic magnetic fields was calculated based on numerical magneto-hydrodynamical (MHD) simulations whose results we summarise in this section (see also Jedamzik & Sigl 2011, for a more recent application).

2.1 Turbulent magnetohydrodynamics

The exceedingly large Prandtl numbers in the early Universe allow one to neglect dissipative effects due to finite conductivity. Further, the generation of primordial magnetic fields in magnetogenesis scenarios is generally believed to occur during well-defined periods (e.g. QCD-transition). Subsequent evolution of these magnetic fields is therefore described as a *free decay* without any further input of kinetic or magnetic energy, i.e. as freely decaying MHD. Due to the largeness of the speed of sound in a relativistic plasma $v_{\rm s} = 1/\sqrt{3}$, the assumption of incompressibility of the fluid is appropriate during most epochs, as well as for a large range of initial magnetic field configurations and energy densities. Exception to the incompressibility may occur for initial conditions which result in magnetic fields of strength $B \gtrsim 6\times10^{-11}$ G (comoving to the present epoch) and of course during the formation of the first stars and galaxies.

Incompressible MHD can be described by the following equations

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v} - (\boldsymbol{v}_{\rm A}\cdot\nabla)\,\boldsymbol{v}_{\rm A} = \boldsymbol{f}\,, \tag{1}$$

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$$\frac{\partial \boldsymbol{v}_\mathrm{A}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v}_\mathrm{A} - (\boldsymbol{v}_\mathrm{A}\cdot\nabla)\,\boldsymbol{v} = \nu\nabla^2\,\boldsymbol{v}_\mathrm{A}\,, \tag{2}$$

where we have defined a *local* Alfvén velocity $\boldsymbol{v}_\mathrm{A}(x) = \boldsymbol{B}(x)/\sqrt{4\pi(\varrho+p)}$, and where $\boldsymbol{v}$, $\boldsymbol{B}$, ϱ and p are the velocity, magnetic field, mass-energy density, and pressure, respectively. Here fluid dissipative terms in the Euler equation are given by

$$\boldsymbol{f} = \begin{cases} \eta\nabla^2\,\boldsymbol{v} & l_\mathrm{mfp} \ll l \\ -\alpha\boldsymbol{v} & l_\mathrm{mfp} \gg l \end{cases}\,, \tag{3}$$

where there exists a distinction between dissipation due to diffusing particles, with mean free path smaller than the characteristic scale $l_\mathrm{mfp} \ll L$, or dissipation due to a free-streaming (i.e. $l_\mathrm{mfp} \gg L$) *background* component exerting drag on the fluid by occasional scatterings with fluid particles. Both regimes are of importance in the early Universe as already noted in Jedamzik et al. (1998). An important characteristics of the fluid flow is given by its local kinetic Reynolds number

$$\mathrm{Re}(l) = \frac{v^2/l}{|\boldsymbol{f}|} = \begin{cases} \dfrac{v\,l}{\eta} & l_\mathrm{mfp} \ll l \\ \dfrac{v}{\alpha\,l} & l_\mathrm{mfp} \gg l \end{cases}\,, \tag{4}$$

with l some length scale. For most magnetic field configurations it is possible to define an *integral scale*, L, i.e. the scale which contains most of the magnetic and fluid kinetic energy. We will also refer to this scale as the *coherence scale* or *coherence length* of the magnetic field. In the case of *turbulent* flow, with $\mathrm{Re}(L) \gg 1$ on this scale, the decay rate of the total energy is independent of dissipative terms and only depends on the flow properties on the integral scale. This is in contrast to the decay of magnetic and fluid energy in the *viscous* regime, $\mathrm{Re}(L) \ll 1$, where the total decay rate depends on the magnitude of viscosities.

2.1.1 Nonhelical fields

Consider Eqs. (1) and (2) with a stochastic, statistically isotropic, magnetic field and, for the purpose of illustration, with initially zero fluid velocities. For the moment we will also assume that the magnetic field does not possess any net helicity. In the limit of large Reynolds numbers on the coherence scale, the dissipative term may be neglected on this scale. Magnetic stresses will establish fluid motions of the order $v \approx \boldsymbol{v}_\mathrm{A}$ within an Alfvén crossing time $\tau_\mathrm{A} \approx l/\boldsymbol{v}_\mathrm{A}$, at which point back reaction of the fluid flow on the magnetic fields will prevent further conversion of magnetic field energy into kinetic energy. The resultant fully turbulent state is characterized by close-to-perfect equipartition (in the absence of net helicity)

$$\langle \boldsymbol{v}^2\rangle \approx \langle {\boldsymbol{v}_\mathrm{A}}^2\rangle\,, \tag{5}$$

between magnetic and kinetic energy.

Non-linear MHD processes quickly establish turbulence on scales below the integral scale (cf. Fig. 1). Working with

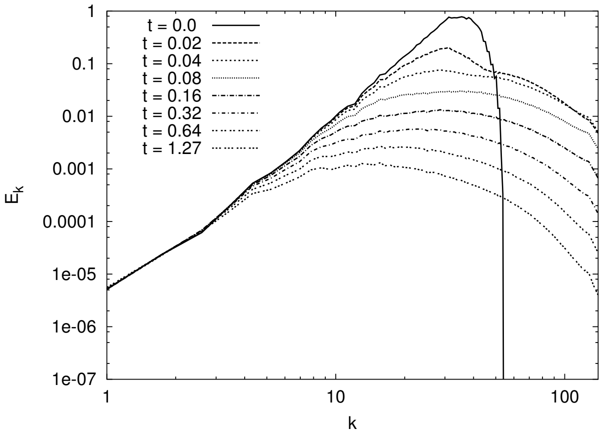

Fig. 1 Evolution of magnetic energy spectra in the turbulent regime for a magnetic field with no initial helicity with $n \approx 4$ (from Banerjee & Jedamzik 2004).

Fourier transforms (assuming statistical isotropy and homogeneity) and defining the total magnetic- and kinetic- energy density

$$\begin{aligned} E &\approx \int \mathrm{d}\ln\,k\,k^3\Big(\langle|v_k|^2\rangle + \langle|v_{\mathrm{A},k}|^2\rangle\Big) \\ &\equiv \int \mathrm{d}\ln\,k\,E_l\,, \end{aligned} \tag{6}$$

one finds that a typical rms-velocity perturbation on scale $l = 2\pi/k$ is $v_l \approx \sqrt{k^3\langle|v_k|^2\rangle} \approx \sqrt{E_l}$. Here we set $(\varrho+p)/2 = 1$, as frequently done in studies of incompressible MHD, such that energy density has the dimension of velocity square. From the Fourier transformed damping equations one sees that the dissipation of energy is dominated by flows on the smallest scales (largest k), given that energy spectra E_l fall not too steeply with growing k. Dissipation of energy into heat thus occurs at some much smaller scale $l_\mathrm{diss} \ll L$ (where $\mathrm{Re}(l) \approx 1$). The transport of the fluid energy from the integral scale L to the dissipation scale l_diss occurs via a cascading of energy from large scales to small scales, referred to as *direct cascade*.

It is known that this cascading of energy occurs as a quasi-local process in k-space, with flow eddies on a particular scale l breaking up into eddies of somewhat smaller scale $\sim l/2$ first described by Kolmogorov (1941). This continuous flow of energy through k-space

$$\frac{\mathrm{d}E_l}{\mathrm{d}t} \approx \frac{E_l}{\tau_l} \approx \mathrm{const}(k), \tag{7}$$

results in a quasi-stationary energy spectrum on scales $l \lesssim L$, with energy flow rates approximately independent of wave vector. Typical energy dissipation times τ_L are given by an eddy-turnover time at the integral scale $\tau_L \sim \tau_\mathrm{eddy} \approx L/v_L \sim L/(\sqrt{E_L})$.

Evolution of global properties of the magnetic field in freely decaying MHD, such as total energy density and coherence length, depend on the magnetic field spectra on scales above the integral scale, $l > L$, and are related to initial conditions. Consider an initial magnetic field with blue spectrum,

$$E_k \approx E_0 \left(\frac{k}{k_0}\right)^n = E_0 \left(\frac{l}{L_0}\right)^{-n} \quad \text{for } l > L_0 \,. \tag{8}$$

The scale-dependent relaxation time, $\tau_l \approx l/v_{\mathrm{A},l} \approx l/\sqrt{E_l}$ (with $v_{\mathrm{A},l} = \sqrt{k^3\langle|v_{\mathrm{A},k}|^2\rangle}$) increases with scale as $\tau_l \propto l^{1+n/2}$. Transfer of magnetic energy to kinetic energy and a fully developed turbulent state may only occur for times $t \gtrsim \tau_l$. When such a state is reached the energy on this scale decays through the cascading of large-scale eddies to smaller-scale eddies down to the dissipation scale. Since the relaxation time for the "next" larger scale l is longer, this larger scale now becomes to dominate the energy density, i.e. becomes the integral- or coherence- scale. This is sometimes referred to as *selective decay* of modes in k-space. The remaining energy density is then the initial energy density of modes between the very largest scales and this next larger scale. Given these arguments and the initial spectrum of Eq. (8) one then may derive for the time evolution of energy and coherence length of the magnetic field

$$\begin{aligned} E &\approx E_0 \left(\frac{t}{\tau_0}\right)^{-\frac{2n}{2+n}}, \\ L &\approx L_0 \left(\frac{t}{\tau_0}\right)^{\frac{2}{2+n}}, \\ &\text{no helicity}, \quad \mathrm{Re} \gg 1 \end{aligned} \tag{9}$$

for $t \gtrsim \tau_0$, where τ_0 is the relaxation time on the scale L_0, i.e. $\tau_0 \approx L_0/\sqrt{E_0} \approx L_0/v^A_{L,0}$, and where indices 0 denote quantities at the initial time. For instance, for a spectral index of $n = 3$ (which corresponds to the large-scale magnetic field due to a large number of randomly oriented and homogeneously distributed magnetic dipoles (Hogan 1983) the energy density follows $E \propto t^{-6/5}$ which is Saffman's law known from fluid dynamics (Lesieur 2008; Saffman 1967).

An increase of magnetic field coherence scale with time due to selective decay may be observed in Fig. 1, whereas the decay of magnetic energy density for a variety of initial magnetic field spectra is shown in Fig. 2. It can be seen that initial spectra with larger n indeed lead to a more rapid decrease of energy with time as predicted by Eq. (9).

2.1.2 Helical fields

So far the evolution of a statistically isotropic and homogeneous magnetic field in the absence of net helicity was considered. Given that magnetic helicity should be an ideal invariant in the early Universe (where the conductivity is almost perfect), and that magnetic fields with even small initial net helicity ultimately reach maximal helicity density

$$\mathcal{H} \lesssim \mathcal{H}_{\max} \approx \langle B^2 L\rangle \approx (8\pi)\, E\, L\,, \tag{10}$$

it is important also study the maximally helical case. A maximally helical state is reached during the course of MHD

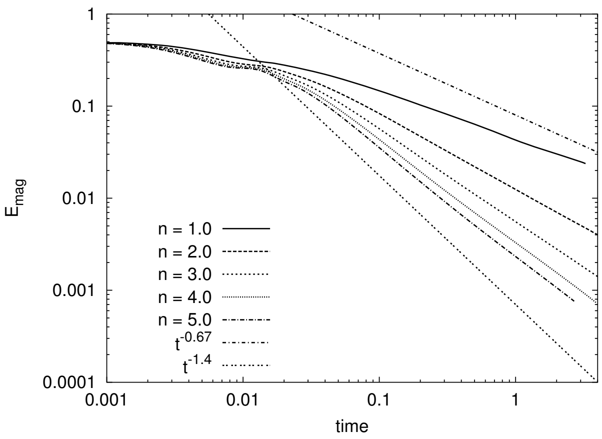

Fig. 2 The evolution of the magnetic energy in the turbulent regime for different initial energy spectra n, where $E_k = k^3\,|b_k|^2 \propto k^n$ with a cut-off $k_{\mathrm{c}} \approx 32$ for the non-helical case. In this case, the damping law depends on the spectral index (cf. Eq. 9). For comparison, the theoretical predicted damping laws for $n = 1$ ($E \propto t^{-0.67}$) and for $n = 5$ ($E \propto t^{-1.4}$) are also shown (from Banerjee & Jedamzik 2004).

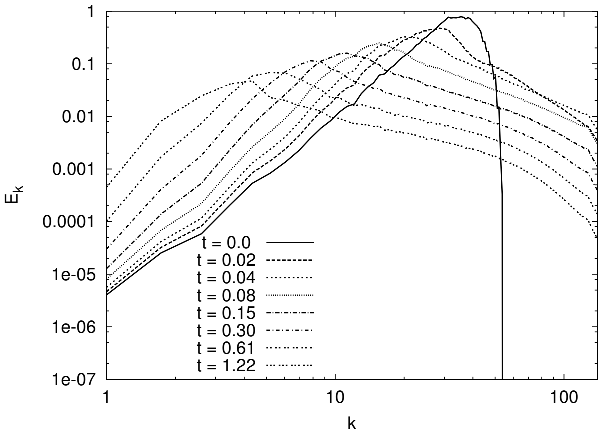

Fig. 3 Evolution of magnetic energy spectra in the turbulent regime for magnetic fields with initially maximal helicity. The spectral index of the energy spectra is $n \approx 4$ (from Banerjee & Jedamzik 2004).

turbulent evolution due to a slower decay of the helical component of fields as compared to the non-helical one. When maximal helicity is reached magnetic field evolution is significantly altered with respect to the case of zero, or submaximal helicity.

Figure 3 illustrates the intriguing property of self-similarity of spectra at different times. This phenomenon of self-similarity has also been observed by Christensson et al. (2001). Magnetic field amplification on very large scales occurs even at times much shorter than the typical relaxation time for magnetic fields (i.e. Alfvén crossing time) on these scales, indicating the topological constraint imposed on the field evolution. If magnetic fields on large scales would not be enhanced, magnetic coherence length could not grow with time, as generally the initially existing energy

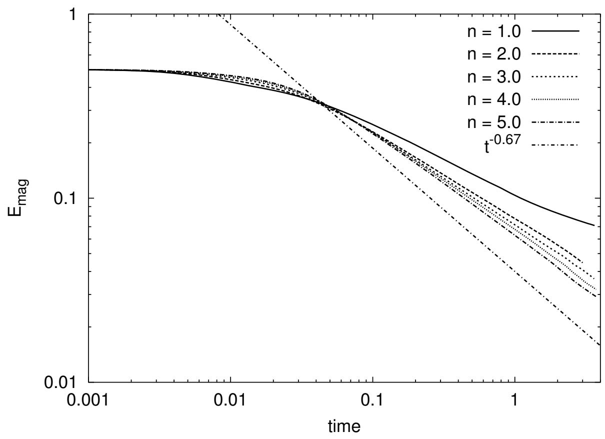

Fig. 4 The evolution of the magnetic energy in the turbulent regime for different initial energy spectra n, where $E_k = k^3 |b_k|^2 \propto k^n$. Here, the initial magnetic field is maximal helical. For comparison, also the theoretical damping law, $E \propto t^{-0.67}$, is shown. In contrast to non helical case, the damping law for a helical magnetic field is nearly independent of the spectral index n for $n < 1$ (from Banerjee & Jedamzik 2004).

density on large scales would not suffice to keep $\mathcal{H}$ constant. Simulations of maximally helical fields with different initial spectral indices n show that though the amplitude of large-scale magnetic field grows with time, the spectral index of the magnetic field configuration on large scales seems to be approximately preserved.

The power-law exponents for the decay of energy and growth of coherence length with time

$$E \approx E_0 \left(\frac{t}{\tau_0}\right)^{-2/3}, \quad L \approx L_0 \left(\frac{t}{\tau_0}\right)^{2/3}, \quad \text{maximal helicity}, \quad \mathrm{Re} \gg 1 \tag{11}$$

for $t \gtrsim \tau_0 \approx L_0/\sqrt{E_0} \approx L_0/v^{\mathrm{A}}_{L,0}$, yielding a predicted decay which is independent of the spectral index of the large-scale magnetic field.

Figure 4 shows the total magnetic energy as a function of time for a variety of maximally helical magnetic fields of different initial spectral index. With the exception of the rather red spectrum $n = 1$, for which the Fourier transform of helicity is not peaked in k-space, the decay of energy seems to be indeed approximately independent of spectral index.

2.2 Viscous magnetohydrodynamics

Magnetic field dissipation in high Prandtl number fluids may also occur in the viscous regime, where kinetic Reynolds numbers are much smaller than unity. Of particular importance to MHD evolution in the early Universe is the case of photons or neutrinos free-streaming over the scales of interest, $l_{\mathrm{mfp}} \gg l$, resulting in a drag force in Eq. (1) with drag coefficient α.

In the terminal velocity regime one finds thus

$$\boldsymbol{v} \approx \frac{1}{\alpha} \left(\boldsymbol{v}_{\mathrm{A}} \cdot \nabla\right) \boldsymbol{v}_{\mathrm{A}}, \tag{12}$$

such that $v_l \approx \boldsymbol{v}_{\mathrm{A},L} \left(\tau_{\mathrm{drag}}/\tau_{\mathrm{A},l}\right) \ll \boldsymbol{v}_{\mathrm{A},l}$ for $\tau_{\mathrm{drag}} \equiv \alpha^{-1} \ll \tau_{\mathrm{A},l}$. This yields a kinetic Reynolds number of

$$\mathrm{Re} \approx \left(\frac{\boldsymbol{v}_{\mathrm{A},l}}{\alpha\, l}\right)^2 \ll 1. \tag{13}$$

Though one would naively expect that at small Reynolds number the total energy gets immediately dissipated due to viscous terms, this is not the case (Jedamzik et al. 1998). For large Prandtl number the energy may only be dissipated via the excitation of fluid motions. Nevertheless, due to the strong drag, such excitation is slow and inefficient, and a system well below equipartition between magnetic- and kinetic- energy results. Since the dissipation rate is proportional to the velocity fluctuations v the net effect of strong fluid viscosities is a delayed dissipation and quasi-frozen-in magnetic fields. Note that in the case of viscous MHD, flows are effectively dissipated on the integral scale, and cascading of energy in k-space is not required. One finds for the energy dissipation rate

$$\frac{\mathrm{d}E}{\mathrm{d}t} \approx \frac{E}{\tau_L} \sim \frac{E^2}{L^2\, \alpha} \tag{14}$$

with $\tau_L \approx L/v_L \sim L^2/\alpha\, E$. Hence the asymptotic power-law for decay of energy density and growth of magnetic field coherence length have the same form as Eq. (9) and Eq. (11), and for the non-helical and helical case, respectively, with τ_0 replaced by $\tau_0^{\mathrm{visc}} \approx \tau^{\mathrm{A}}_{L,0} \left(\tau^{\mathrm{A}}_{L,0}/\tau_{\mathrm{drag}}\right) \approx L_0^2 \alpha / E_0$.

2.3 Evolution in the early Universe

The evolution of a stochastic magnetic field in the early Universe is described by alternating epochs of turbulent MHD and viscous MHD. Here the latter epochs occur when viscosities due to neutrinos, or photons, become significant. Such a picture has already been established by Jedamzik et al. (1998). Following the arguments in the Sect. 2.1 the instantaneous integral scale is given by the equality between cosmic time and eddy turnover time at the scale L

$$\frac{1}{t_{\mathrm{eddy}}} \approx \frac{v(L)}{L_{\mathrm{p}}(T)} \approx H(T) \approx \frac{1}{t_H}. \tag{15}$$

In the above expression the subscript p denotes the proper (as opposed to comoving) value of the integral scale, $v(L)$ is the the fluid velocity on scale L, and H is the Hubble parameter. The velocity in the turbulent regime is $v(L) \approx \boldsymbol{v}_{\mathrm{A}}(L)$ and in the viscous regime $v(L) \approx \boldsymbol{v}^2_{\mathrm{A}}(L)\, L/\eta$ and $v(L) \approx \boldsymbol{v}^2_{\mathrm{A}}(L)/\alpha\, L$, in the photon (neutrino) diffusive and free-streaming viscous cases, respectively.

Figures 5 and 6 shows examples for the growth of $L(T)$ for a number of scenarios of magnetogenesis at the EW and QCD phase transitions, respectively (see also Jedamzik & Sigl 2011, for a similar calculation). The evolution is observed as an alternation between turbulent MHD and

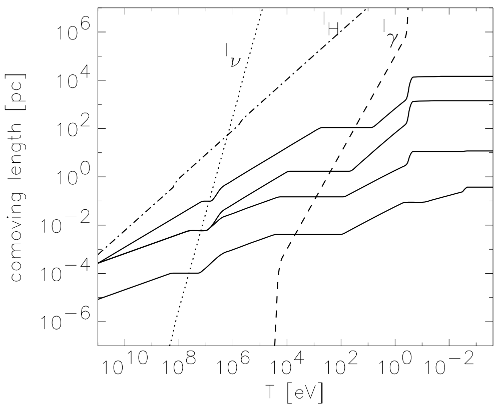

Fig. 5 The evolution of comoving coherence length for initial magnetic field configurations with different spectral indices n and initial magnetic helicities. Solid lines from top to bottom: $h_{\rm g} = 1$, $r_{\rm g} = 0.01$; $h_{\rm g} = 10^{-3}$, $n = 3$, $r_{\rm g} = 0.01$; $h_{\rm g} = 0$, $n = 3$, $r_{\rm g} = 0.01$; $h_{\rm g} = 0$, $n = 3$, $r_{\rm g} = 10^{-5}$. The labels l_ν, l_γ, l_H refer to the comoving mean free paths of neutrinos and photons and the comoving Hubble length, respectively. The epoch of magnetogenesis was assumed to occur during the electroweak phase transition ($T_{\rm g} = 100\,$GeV) (from Banerjee & Jedamzik 2004).

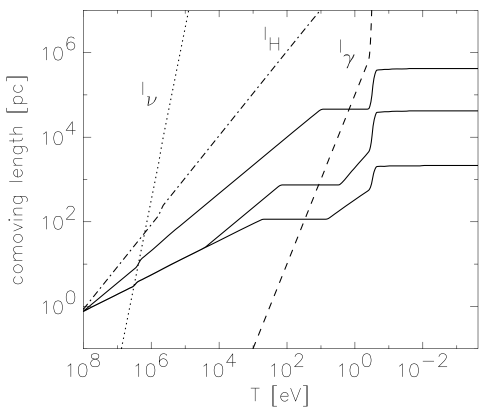

Fig. 6 The evolution of comoving coherence length for different initial magnetic field configurations. Solid lines from top to bottom: $h_{\rm g} = 1$, $r_g = 0.083$, $n = 3$; $h_g = 10^{-3}$, $r_g = 0.083$, $n = 3$; $h_g = 0$, $r_g = 0.083$, $n = 3$. The epoch of magnetogenesis was assumed to occur during the QCD phase transition ($T_{\rm g} = 100\,$MeV) (from Banerjee & Jedamzik 2004).

viscous MHD. "Viscosity" here is early on due to neutrinos, some time before recombination due to photons, and after recombination due to hydrogen-ion scattering and hydrogen-hydrogen scattering. Particularly notable are phases where the growth of $L(T)$ is halted completely. This occurs either at epochs before recombination in the viscous regime with diffusing photons or neutrinos, as well in part of the regime when those particles are free-streaming or at epochs after recombination. However, the growth of $L(T)$ and concomitant decrease of $B(T)$ during the late phases of viscous MHD with free-streaming photons (neutrinos) may be faster than the growth of $L(T)$ during turbulent MHD. Those initial conditions lead to relatively strong

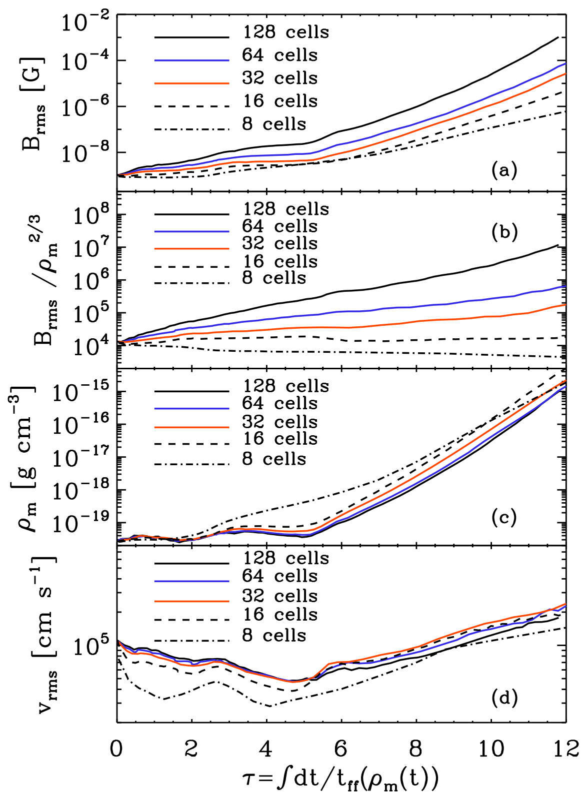

Fig. 7 Evolution of the dynamical quantities as a function of $\tau = \int \mathrm{d}t/t_{\rm ff}$, for five runs with different number of cells to resolve the local Jeans length. (*a*): the rms magnetic field strength $B_{\rm rms}$, amplified to 1 mG from an initial field strength of 1 nG, (*b*): the evolution of $B_{\rm rms}/\varrho_{\rm m}^{2/3}$, showing the turbulent dynamo amplification by dividing out the maximum possible amplification due to pure compression of field lines, (*c*): the evolution of the mean density $\varrho_{\rm m}$, and (*d*): the rms velocity $v_{\rm rms}$. The onset of runaway collapse commences at about $\tau \sim 6$ (from Sur et al. 2010).

magnetic fields at recombination and result in a rapid increase of $L(T)$ at $T_{\rm rec} \approx 0.3\,$eV, whereas for weaker fields $B \lesssim 10^{-13}\,$G a similar jump occurs at reionization. Here, effects due to structure formation are not taken into account here (but see Sect. 3 below).

3 Amplification by the small-scale turbulent dynamo

So far we have discussed the damping of magnetic fields due to excitations of fluid motions in high Reynolds number plasma in the case of relatively strong magnetic fields, i.e. for magnetic fields which energy is comparable to the kinetic energy of the cosmic gas. Otherwise, it is fairly unclear how strong magnetic fields will be briefly after magnetogenesis as those scenarios are highly model dependent (see e.g. the reviews Kandus et al. 2011; Widrow et al. 2012). Nevertheless, if those primordial magnetic fields are very weak (compared to the kinetic motions) they will undergo strong amplification due to the small-scale dynamo as long as the medium is turbulent. This was first pointed out Batchelor (1950) and analytically analysed by Kazantsev (1968).

The properties of the small-scale dynamo have been explored both in computer simulations of driven turbulence without self-gravity and in analytic models (Brandenburg & Subramanian 2005; Federrath et al. 2011a; Haugen et al. 2004; Schekochihin et al. 2004) as well as in the context of magnetic fields during the formation of the first galaxies and galaxy clusters (Latif et al. 2013; Sur et al. 2010, 2012; Xu et al. 2009). Analytic estimates show that the small-scale dynamo could be important already during the radiation dominated era (Wagstaff et al. 2013) and during the formation of the first stars and galaxies (Arshakian et al. 2009; de Souza & Opher 2010; Schleicher et al. 2010; Schober et al. 2012b).

In general one expects that weak magnetic fields will be exponentially amplified with a growth rate $\Gamma \propto \mathrm{Re}^{1/2}$ (in the case of Kolomogorov type turbulence, see e.g. Schober et al. 2012a,c). During the collapse of a primordial gas cloud gravitational compression can at most lead to an amplification of the magnetic field strength by a factor of $\sim \varrho^{2/3}$ in the limit of perfect flux freezing (i.e., ideal MHD). Any stronger increase implies the presence of an additional amplification mechanism. The results of the collapse simulations by Sur et al. (2010) are summarised in Fig. 7 where one clearly observes field amplification by the small-scale turbulent dynamo. Note that the amplification gets stronger for higher resolution, i.e. for increasing Reynolds numbers, as predicted by the Kazantsev theory (see also Federrath et al. 2011b; Haugen et al. 2004).

4 Conclusions

Taken all together, the above results strongly indicate that dynamically important magnetic fields will be present at very early stages during cosmic evolution, hence they can not be neglected in modelling structure formation and the subsequent evolution. Nevertheless, further knowledge and constraints on the magnetic fields strength and its coherence length during the various epochs in the early universe is necessary to fully understand the impact of magnetic fields.

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