From central stars of planetary nebulae to white dwarfs

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Abstract

The final stages of the evolution of low and intermediate mass stars are discussed. We review how stars evolve from the second (asymptotic) giant branch through the planetary nebula phase to the white dwarf cooling sequence. We concentrate on the spectroscopic aspects and which conclusions can be drawn from this information. About 3/4 of all white dwarfs and central stars of planetary nebulae are hydrogen-rich. The remaining quarter has lost its entire hydrogen envelope and exhibits now a helium-rich surface.

We give evidence that, in contrast to previous claims, there exists a continuous hydrogen-rich sequence from the central stars to the white dwarfs. However, a mysterious gap is present in the helium-rich sequence, the so-called DB-gap. This is possibly explained by a very thin layer of hydrogen hiding the helium during this particular phase and mixed into the helium envelope, nearly undetectable by spectroscopic techniques, during other parts of the white dwarf sequence. We discuss stars at the hot and cool edges of the DB gap, which possibly are on their way from the hydrogen-rich to the hydrogen-poor sequence or back.

1 Introduction

White dwarfs are the burnt out remnants of low and intermediate mass stars. Their gravitational contraction is stopped by the pressure of the degenerate electron gas in their interior. Since low and intermediate stars outnumber the massive stars by orders of magnitude, most stars will finally end up as white dwarfs. In this review we will focus on the formation of white dwarfs and their early evolution.

The evolution of low and intermediate mass stars from the first ignition of the nuclear fusion of hydrogen (usually dubbed "hydrogen burning") on the main sequence to their final stage is now basically well understood. Figure 1 shows the evolution of a $3M_{\odot}$ star in the temperature-luminosity plane, the theoretical version of the famous Hertzsprung-Russell diagram. The stars begin their evolution after the ignition of hydrogen in their center on the zero-age-main-sequence (ZAMS). After the core hydrogen burning main sequence phase hydrogen fusion ignites in a shell around the helium core and the star becomes a red giant (first red giant branch – RGB). This stage ends when the helium core has become massive enough to sustain helium burning. This phase can be identified with the horizontal branches observed in globular clusters.

When the helium core is exhausted, a helium burning shell is formed and the star becomes a red giant for the second time. This phase is called asymptotic

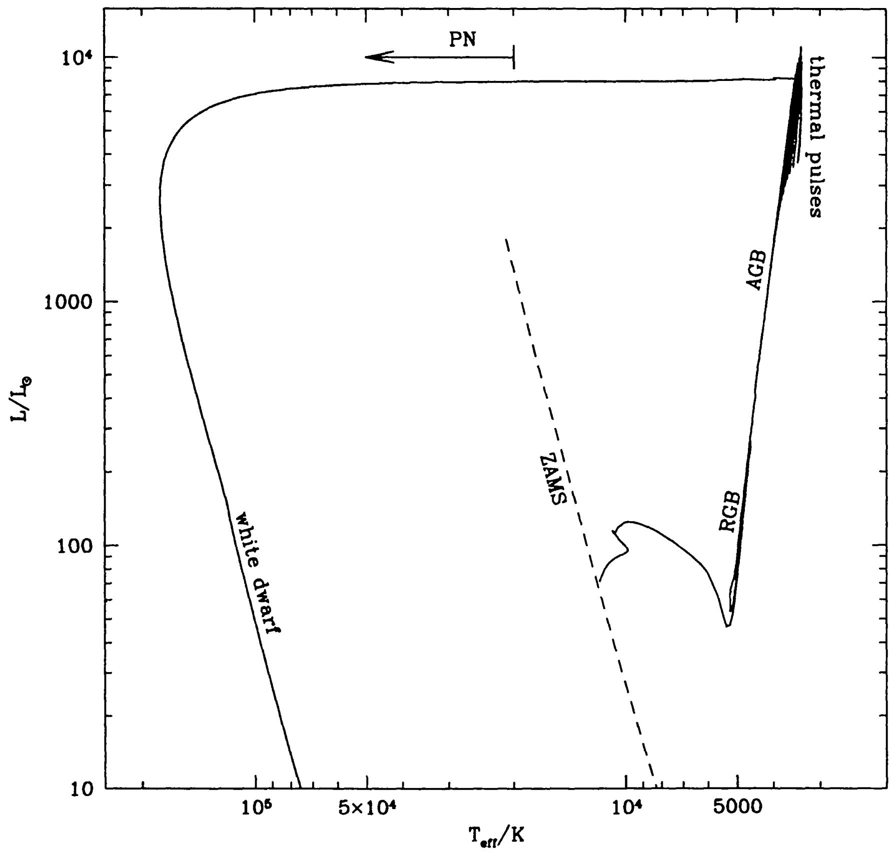

Figure 1: The evolution of a $3M_{\odot}$ star as calculated by Blöcker (1995a,b). Important stages are labeled. This star ends as a $0.625M_{\odot}$ white dwarf. The dashed line indicates the position of the ZAMS.

giant branch (AGB) stage. AGB stars consist of a carbon and oxygen composed core, produced by helium fusion, surrounded by helium and hydrogen burning shells. As the star evolves towards the tip of the AGB both nuclear burning shells start to interact: only one shell is burning at a time. While the hydrogen shell is burning smoothly, the helium burning ignites violently with a very high energy production (helium shell flash). As a result the outer region of the star expands and the hydrogen burning shell is extinguished. After some time the helium burning stops, because its nuclear fuel is consumed, and hydrogen burning reignites. This phenomenon is known as thermal pulses. These pulses are repeated several times during the AGB phase.

The outer part of the star is formed by a convective envelope of essentially unprocessed material. Due to the enormous size of AGB stars material at the stellar surface is only loosely bound and a combination of radiation pressure and Mira type pulsations causes a strong stellar wind. Mass loss rates up to $10^{-4}M_{\odot}$/year are observed. This reduces the mass of the stellar envelope rapidly. When the envelope mass is reduced below some hundredths solar masses the convection zone, which extends over most of the stellar radius, cannot be sustained any more and the star starts to shrink at constant luminosity. This results in a fast increase of the surface temperature. Maximum effective temperatures ($T_{\rm eff}$) of 100000 K and more are reached. After $T_{\rm eff}$ exceeds approximately 20000 K the stellar UV flux is strong enough to ionize hydrogen in the surrounding material lost during the AGB phase. The circumstellar matter starts to radiate light – a planetary nebula (PN) is born.

Finally the hydrogen and helium fuel is exhausted and nuclear fusion in the shell sources ceases. This corresponds roughly to the hottest point reached during the evolution of the central star of the PN. Now, energy production has stopped and a white dwarf with an electron degenerate core has been produced. The final fate of a white dwarf is to shrink and cool down along the so-called cooling sequence. The coolest (and oldest) white dwarfs, which are observed, have surface temperatures of $\approx$4000 K. The mass loss on the AGB is efficient enough that even a star with an initial mass of $8 \pm 2 M_\odot$ (Weidemann & Koester 1983) becomes a white dwarf with a mass below the Chandrasekhar limit of $1.4 M_\odot$. White dwarfs with higher masses are not stable (Chandrasekhar 1939) and a burnt out stellar remnant with a higher mass will collapse to a neutron star

So far, we have concentrated on the internal evolution. What do we know about the outer layers, which are the only part accessible to direct observations? The spectral evolution is discussed in Sect. 2. We focus on the transition from central stars of planetary nebulae (CPNe) to the white dwarfs in Sect. 3. Observations which should help to understand the astonishing spectral evolution of white dwarfs are described in Sect 4. We finish with some conclusions.

2 Spectral evolution

2.1 Central stars of planetary nebulae

Most central stars show spectra, which are similar to that of normal hot B and O type stars (see Fig. 2). Although some elements may be enriched or depleted by dredge up processes the hydrogen and helium content remains virtually unmodified.

However, about every fifth of all central stars displays a spectrum that is completely different. No hydrogen can be detected in their atmospheres. Their spectra show strong helium and carbon lines. Representatives are the Wolf-Rayet type central stars, which have a strong stellar wind, producing pronounced emission lines (see spectrum in Fig 2) and the PG 1159 stars, which mark the transition from the central stars to the white dwarfs. The latter class of stars is defined by showing a broad He II/C IV absorption trough at 4686 Å. The class is named after the prototype PG 1159+035, discovered in the Palomar-Green survey (Green et al. 1986). A spectrum of a PG 1159 central star is shown in Fig. 6.

The mere existence of hydrogen-poor CPNe and white dwarfs is a challenge for stellar evolution theory. Standard theory predicts a H-rich surface for all post-AGB stars. A possible explanation is offered by the so-called born again scenarios in which a star, which has already left the AGB and became a central star or even a white dwarf, suffers a late helium shell flash (Schönberner 1979). Since the outer envelope contains much less mass than AGB envelopes do, the outcome of a thermal pulse is more dramatic than it is on the AGB. A star suffering from a late helium flash returns to the AGB and repeats the post-AGB evolution. The hydrogen envelope can eventually be removed under certain circumstances (Iben 1984; Iben & MacDonald 1995) and the star may become a hydrogen-poor Wolf-Rayet or

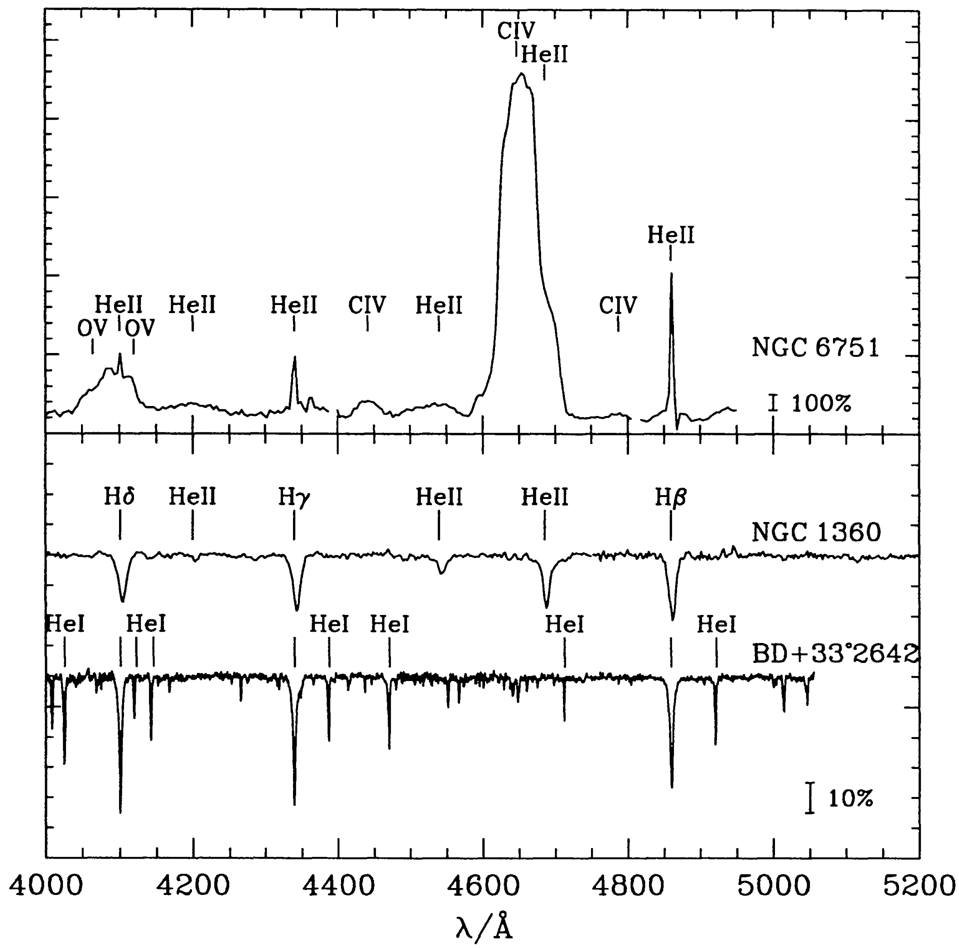

Figure 2: Central star spectra. BD+33°2642 is a cool (20200 K; Napiwotzki et al. 1994) and NGC 1360 a hot (110000 K; Hoare et al. 1996) hydrogen-rich central star. NGC 6751 is an example of a hot helium-, carbon-, and oxygen-rich Wolf-Rayet central star from Koesterke & Hamann (1997). The spectra are normalized to the continuum. Note the different scales of both plots.

PG 1159 star. Comparisons of this scenario with observations and a discussion of possible drawbacks can be found in Tylenda & Gorny (1993) and Napiwotzki et al. (1996).

2.2 White dwarfs

Similar spectral sequences are found for the white dwarfs. However, their separation into a hydrogen-rich and a helium-rich sequence is stricter than for the CPNe. The hydrogen-rich white dwarfs are classified DA. Their optical spectra show broad Balmer lines and nothing else. The line broadening is caused by the high atmospheric pressure present in white dwarf atmospheres. However, line broadening alone cannot explain the absence of lines of elements other than hydrogen. Helium lines would be easily detectable in hot DA white dwarfs for helium abundances as low as 1/100 of the solar value. Thorough analyses revealed that a typical DA atmosphere consists of pure hydrogen. Only in the hottest DA stars ($T_{\rm eff} > 50000$)

detectable metal traces are present. The reasons why white dwarf atmospheres show this high degree of purity is discussed below.

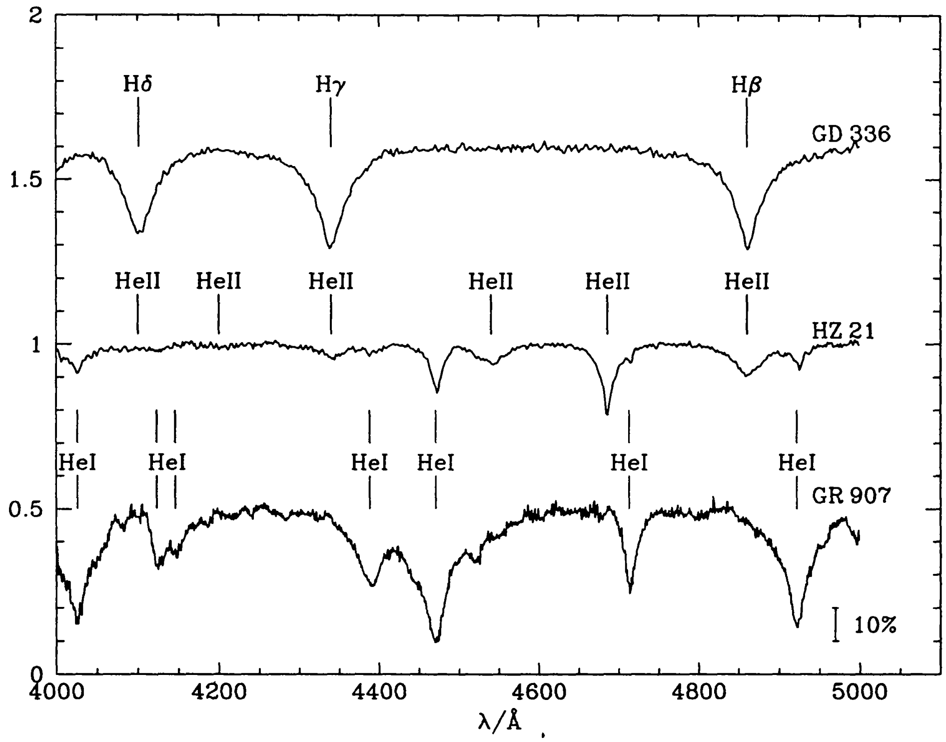

Figure 3: White dwarf spectra. The hydrogen-rich white dwarfs are represented by the DA GD 336, the helium-rich classes by the DO HZ 21, and the DB GR 907. Important lines are marked.

The helium-rich sequence is represented by the hot DO white dwarfs, displaying He II lines, and the cooler DB white dwarfs, showing neutral helium lines. Analogous to the case of the hydrogen-rich DA white dwarfs, the atmospheres of typical DO and DB white dwarfs consist of virtually pure helium.

White dwarfs are compact stars with high surface gravities ($g \approx 10^8 \mathrm{cm/s^2}$). This causes a sedimentation of the heavy elements. Gravitational settling has been identified as reason for the mono-elemental composition of white dwarf atmospheres by Schatzmann (1958). This process is commonly termed diffusion. It is very slow. Diffusion velocities are of the order of 10^{-2} to 10^{-4} cm/s in the outer layers of white dwarfs (Koester 1989). Due to the small velocities there are many processes which can potentially counteract diffusion:

- mass loss,
- accretion,
- convection,
- radiative levitation,
- meridional circulation.

Mass loss: It is well known that CPNe can drive heavy mass loss. When the stars shrink to white dwarf dimensions, mass loss rates drop below the detection limit. However, a very small amount of mass loss can effectively prevent the sedimentation of heavy elements. Detailed calculations of diffusion in the presence of mass loss carried out by Michaud (1987), Chayer (1997), and Unglaub & Bues (1998) derived critical rates of $\dot{M} = 10^{-14} \ldots 10^{-15} M_{\odot}/\mathrm{yr}$ at which diffusion becomes ineffective. Such low mass loss rates are virtually undetectable. Indirect evidence comes from blueshifted metal lines found in the spectra of some hot white dwarfs. They can be interpreted as the fingerprint of circumstellar material of these white dwarfs. However, this interpretation remains controversial (cf. e.g. Holberg 1998 and Dreizler et al. 1998).

Accretion: The critical accretion rate is of the same order as the critical mass loss rate. It is not expected that this rate is reached by an isolated hot white dwarf. The situation might be different for cool white dwarfs, which will not be considered here.

Meridional circulations: Eddington (1925) and Vogt (1925) have shown that the rotation of a star should lead to a large scale flow, termed meridional circulation. However, white dwarfs are very slow rotators. In fact, spectroscopic methods have up to now not been able to detect any evidence of rotation (Pilachowski & Milkey 1984, Koester & Herrero 1988, Heber et al. 1997). Therefore we can rule out any significant influence of rotation on the surface composition of typical white dwarfs.

Convection: The flow velocities in a convection zone are much higher than diffusion velocities can be. Thus one can assume a homogeneous mixture within the convection zone. Diffusion might only play a role, if it causes a different composition at the base of the zone, where the time scales are much larger than inside the atmosphere. The envelopes of hydrogen-rich DA white dwarfs are stable against convection down to $T_{\mathrm{eff}} \approx 15000\,\mathrm{K}$. However, the helium envelope of DB stars is unstable, which probably plays a major role for their spectral evolution.

Radiative levitation: Photons transfer momentum on absorbing atoms, which causes radiation pressure. This can selectively support certain elements against gravitational settling in hot white dwarfs with very strong radiation fields (Vauclair et al. 1979).

As the white dwarfs cool down the mass loss ceases and the radiative levitation becomes unimportant. When the elements are no longer radiatively supported, the strong gravitational forces will finally cause a settling of all heavy elements on short timescales. Only the lightest element (hydrogen or helium respectively) remains visible.

2.3 Gaps

In summary we know two apparently distinct spectral sequences of central stars and white dwarfs: a hydrogen-rich and a hydrogen-deficient (helium- and carbon-rich) one. For the hydrogen-deficient CPNe there exists a continuous sequence from the luminous central stars (Wolf-Rayet stars of spectral type [WC]) to the low luminosity DO white dwarfs via the PG 1159 stars, hot pre-white dwarfs with typical temperatures of 100000 K. Méndez (1991) presented a compilation of spectral types for (mostly luminous) CPNe and found a fraction of about 1/3 hydrogen-deficient objects. Liebert (1986) reported a ratio of H-rich to H-poor stars of (7±3):1 for the hot white dwarfs (spectral types DA and DO resp.) in the Palomar-Green survey (Green et al. 1986), but noted a lack of very hot ($T_{\rm eff} > 70000$ K) DA white dwarfs (the counterparts of the PG 1159 stars).

Besides this "hot DA gap" another gap is know in the helium-rich white dwarf sequence. The coolest DO white dwarf known has a temperature of 48000 K, while the hottest known DB has $T_{\rm eff} \approx 30000$ K. In between no helium-rich white dwarf is known. This "zone of avoidance" is termed DB gap.

Fontaine & Wesemael (1987) discussed the DB gap and the apparently lack of very hot H-rich white dwarfs and proposed that (nearly) all CPNe finally loose their hydrogen-rich surface layer and evolve through one channel: the PG 1159/DO stars. After the stars have become white dwarfs gravitational settling causes small traces of hydrogen, hidden in the helium layer, to float up and form a very thin H-layer (therefore this hypothesis is known as very thin layer scenario). When the white dwarf cools down to ≈30000 K the helium layer beyond the surface becomes convectionally unstable. If the hydrogen layer is thin enough, the hydrogen can be mixed into the helium and becomes invisible again. White dwarfs with a thicker hydrogen layer will remain DA. Although this scenario is in contradiction to theoretical calculations (e.g. Schönberner 1983), which predict a "thick" H-layer of $\approx 10^{-4} M_{\odot}$, it yields a natural explanation of the DB gap and the lack of hydrogen-rich very hot pre-white dwarfs. In the next section we will present the results of a spectroscopic survey of central stars of old PNe and will use this for a rediscussion of the spectral evolution of pre-white dwarfs.

3 Spectra of central stars of old PNe and their analysis

3.1 Sample selection and classification

Due to their faintness the number of central stars of old PNe with a spectral classification remained very small (cf. Méndez 1991; Acker et al. 1992). The first systematic survey was carried out by Napiwotzki & Schönberner (1995). We selected apparently very old, faint and extended nebulae whose central star could be identified. The latter point is not trivial, because these central stars are faint, the nebulae have a large angular size, and the central stars are not necessarily located in the center of the nebulae (cf. the case of Sh 2-174 in Fig. 5). An important contribution to this sample comes from the still ongoing discoveries of old PNe by Weinberger and collaborators (see Weinberger 1977, Kerber et al. 1996 and references therein). Many central star identifications were taken from these

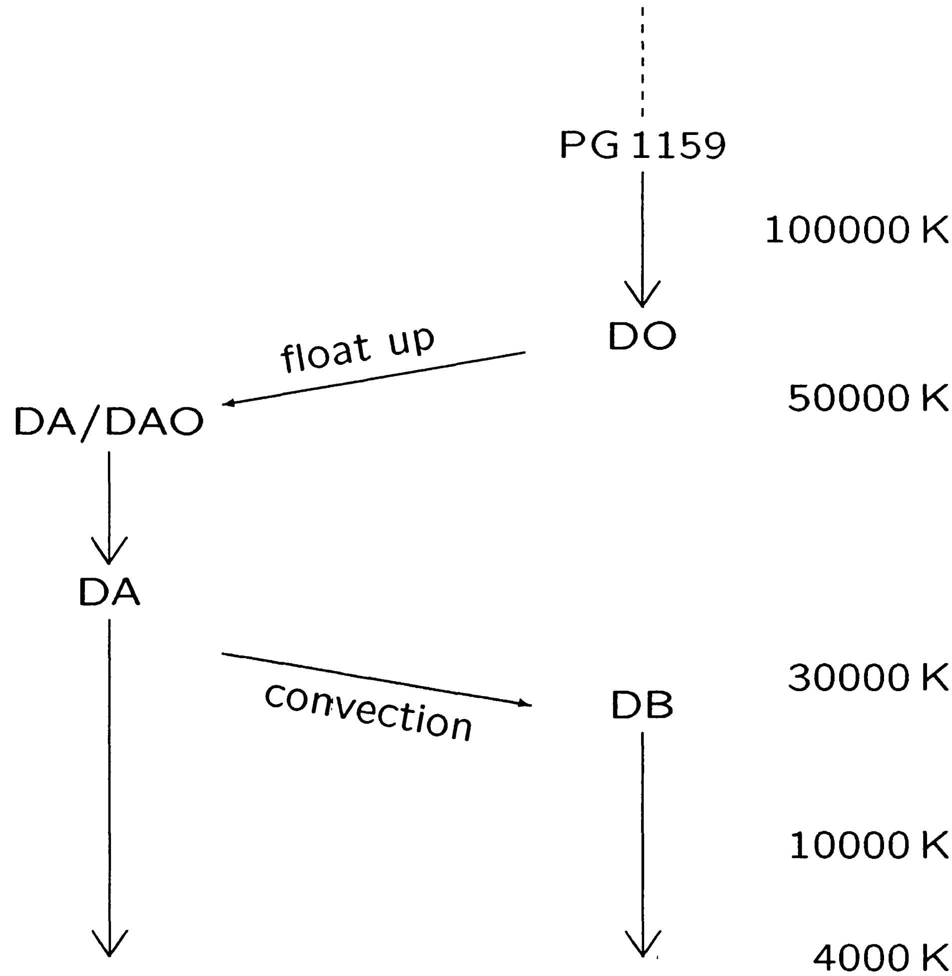

Figure 4: Fontaine & Wesemael (1987) scenario for the spectral evolution of CPNe and white dwarfs.

sources or from Kwitter et al. (1988). Almost all PN included in the Napiwotzki & Schönberner sample have ages in excess of 10000 years. Due to the ongoing nebular expansion the densities are usually very low ($n < 10\,\mathrm{cm}^{-3}$), not much denser than the surrounding interstellar matter (ISM). If the central star and its PN have a peculiar velocity with respect to the ISM, the nebula shape can be highly distorted by the interaction with the ISM (Borkowski et al. 1990). A spectacular example is the PN Sh 2-174 (Napiwotzki & Schönberner 1993; Tweedy & Napiwotzki 1994): the motion of the nebula is slowed down by the ISM ram pressure, while the central star (GD 561) is not influenced. As the result the central star is now seen at the outer rim of the PN (Fig. 5)!

Napiwotzki & Schönberner (1995) classified a total of 38 central stars of old PNe. Most of the observed objects have high surface gravities, many are white dwarfs. About three quarters of the CPNe display H-rich atmospheres. Seven new members of the rare class of helium and carbon dominated PG 1159 stars were discovered. Three stars belong to the previously unknown class of hybrid central stars (Napiwotzki & Schönberner 1991, Napiwotzki 1992). They display a He II/C IV absorption trough typical for PG 1159 stars, but strong Balmer lines, too. Sample

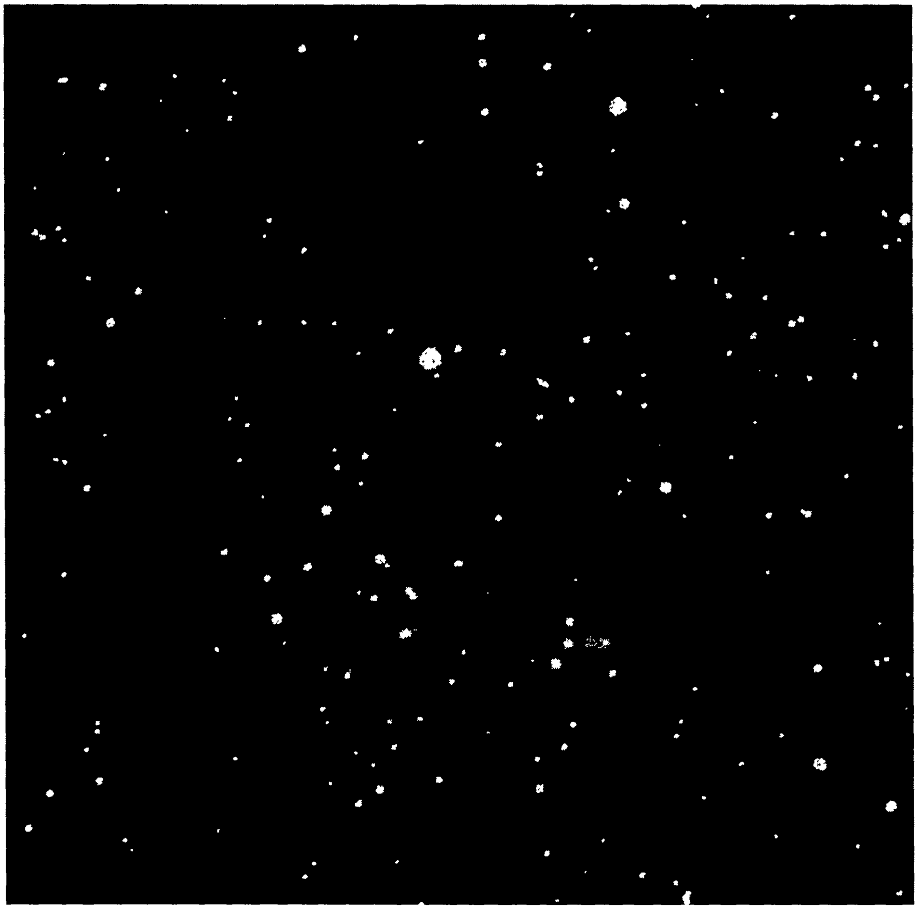

Figure 5: The planetary nebula Sh 2-174 in the light of the [N II] 6584 Å emission line (Tweedy & Napiwotzki 1994). The position of the "central" star GD 561 is marked by the arrow.

spectra of stars of these three classes are shown in Fig. 6. The remaining three stars are close binary CPNe. A complete list is given in Napiwotzki & Schönberner (1995). The ratio of H-rich to -poor objects (4:1) is in reasonable agreement with the findings of Liebert (1986) and Méndez (1991). This is not expected from the one channel scenario of Fontaine & Wesemael (1987), but supports the idea of two continuous spectral sequences.

3.2 Spectral analysis

Since our aim was to investigate the properties of the hydrogen-rich sequence, we will restrict ourselves to the analysis of the hydrogen-rich central stars. Analyses of hydrogen-poor Wolf-Rayet central stars were carried out by Leuenhagen et al. (1996) and Koesterke & Hamann (1997). Reviews of the properties of PG 1159 stars are given by Dreizler et al. (1995) and Werner et al. (1997). Before the start of the survey by Napiwotzki (1992) only three analyses of high gravity CPN were published: A 7 (Méndez et al. 1981), NGC 7293 (Méndez et al. 1988a), EGB 6 (Liebert et al. 1989). The much enlarged number of analyses presented in Napiwotzki (1995,

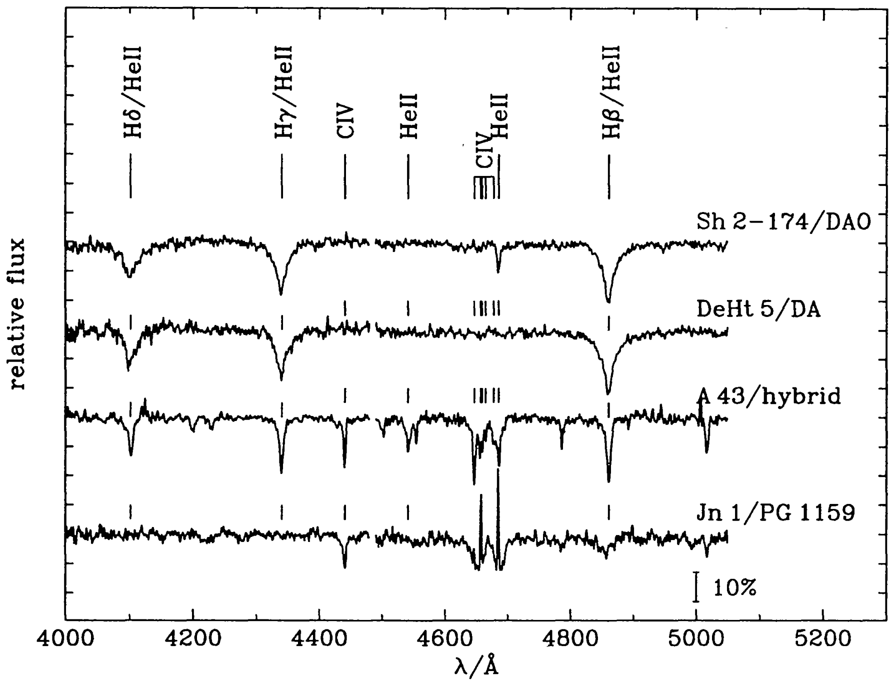

Figure 6: Sample spectra of the central stars of Sh 2-174 (DAO), DeHt 5 (DA), A 43 (hybrid-PG 1159), and Jn 1 (PG1159). Important spectral lines are marked

1998) now allows a more meaningful investigation of the stellar evolution in this region of the HR diagram.

The central stars were analyzed with a grid of hydrogen and helium composed model atmospheres. The analysis of white dwarfs is usually the domain of model atmospheres in LTE (local thermodynamical equilibrium). It is assumed that deviations from LTE are kept small by the high densities of white dwarf atmospheres. Model atmospheres, which drop the LTE assumption, are somewhat loosely called non-LTE (NLTE) atmospheres.

A comparison of LTE and NLTE model spectra with parameters of typical hot DA stars ($T_{\rm eff} = 30000\,{\rm K}$, $\log g = 8.0$) in Fig. 7 shows that LTE is a good approximation in this case: both model spectra are virtually indistinguishable. Only the core of the Hα line shows a significant deviation (hardly visible in Fig. 7). Figure 8 demonstrates that the situation is completely different for typical white dwarf central stars ($T_{\rm eff} = 100000\,{\rm K}$, $\log g = 6.5$). The higher temperature and lower gravity causes large deviations between both types of model spectra. Therefore it is necessary to use NLTE model atmospheres for the analysis of these stars. A complete discussion of this topic can be found in Napiwotzki (1997).

Computing models in NLTE means that we replace the use of the Boltzmann and Saha equations for the computation of the ionization degree and occupation numbers by the complete set of atomic rate equations. These describe for a given species for every level i of every ionization stage the equilibrium of all populating and depopulating processes from all other levels j:

$$n_i \sum_{i \neq j} P_{ij} - \sum_{j \neq i} n_j P_{ji} = 0 \,. \qquad (1)$$

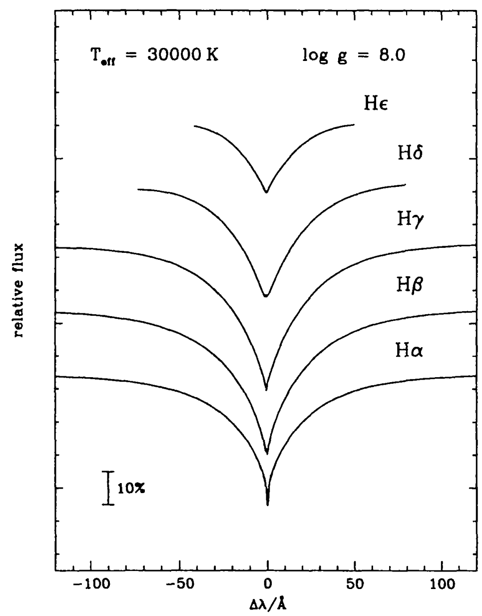

Figure 7: Balmer line profiles computed from NLTE (solid line) and LTE (dashed line) model atmospheres. The parameters are typical for a hot DA white dwarf ($T_{\rm eff} =$ 30000 K, $\log g = 8.0$, pure hydrogen)

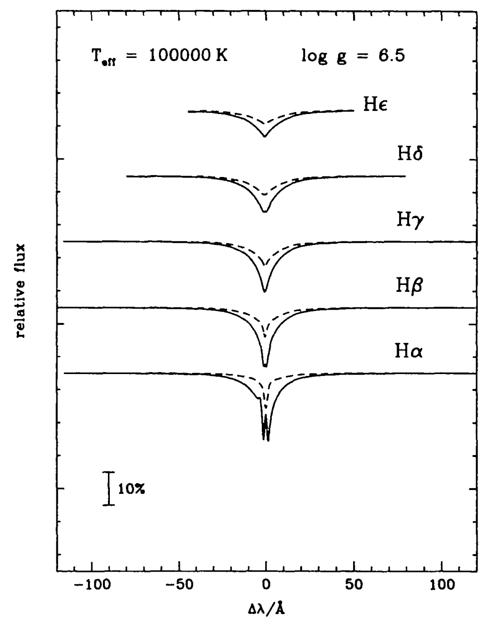

Figure 8: Comparison of NLTE (solid line) and LTE (dashed line) line profiles for typical white dwarf central star parameters ($T_{\rm eff}$ = 100000 K, $\log g$ = 6.5, $n_{\rm He}/n_{\rm H} = 0.01$).

The rate coefficients P_{ij} consist of a radiative component R_{ij} (line transitions, photoionization) and a collisional component C_{ij} (collisional excitation and ionization):

$$P_{ij} = R_{ij} + C_{ij}\,. \tag{2}$$

Additionally we have to solve the equation of radiative transfer

$$J_\nu(\tau_\nu) = \Lambda_{\tau_\nu} S_\nu \tag{3}$$

at every depth and frequency point. J_ν is the mean intensity and S_ν the local source function. The Operator Λ is defined as

$$\Lambda_\tau f = \frac{1}{2}\int_0^\infty f(\tau')E_1(|\tau' - \tau|)d\tau' \tag{4}$$

with E_1 being the first exponential integral.

This coupled set of equations is solved iteratively. We use the ALI (approximate lambda iteration) code developed by Werner (1986). The lambda iteration splits the iteration in an inner and an outer cycle. The outer cycle iterates the radiation field. Formally a new intensity can be computed from the old source function of the last iteration as

$$J_\nu(\tau_\nu)^{\rm new} = \Lambda_{\tau_\nu} S_\nu^{\rm old}\,. \tag{5}$$

The inner cycle solves the rate equations (1) for the actual radiation field. The basic idea of the ALI method is to substitute the exact Λ operator by an approximate lambda operator Λ^*. Equation (5) can be modified to

$$J_\nu(\tau_\nu)^{\rm new} = \Lambda^* S_\nu^{\rm new} + (\Lambda_{\tau_\nu} - \Lambda^*)S_\nu^{\rm old}\,. \tag{6}$$

This means the new mean intensity can be computed by applying Λ^* on the actual source function S_ν^{new}. The correction term $(\Lambda_{\tau_\nu} - \Lambda^*)S_\nu^{\mathrm{old}}$ guarantees the exact solution of equation (3) in the limit of convergence, independent of the choice of Λ^*. The advantage of the ALI method is, that we can choose a Λ^* operator, which minimizes the computational effort and allows fast convergence.

The basic assumptions of the NLTE model atmospheres, calculated with the ALI code developed by Werner (1986) are plane-parallel geometry, hydrostatic and radiative equilibrium. To keep the computational effort within resonable limits we neglected the influence of heavy elements and assumed a hydrogen and helium composition. Elaborate hydrogen and helium model atoms were used for this purpose and pressure dissolution of the higher levels is included according to the Hummer & Mihalas (1988) occupation probability formalism. More details on the model atmospheres are given by Napiwotzki (1997).

Due to the lack of other temperature indicators both, effective temperature and gravity, must be determined from the Balmer lines. For DA white dwarfs this was successfully done by simultaneous line profile fitting of all available Balmer lines (e.g. Bergeron et al. 1992; Napiwotzki et al. 1993). Generally the observed line profiles are well reproduced by model spectra with the optimum parameters. Fig. 9 shows the fit of a typical white dwarf from Napiwotzki et al. (1998).

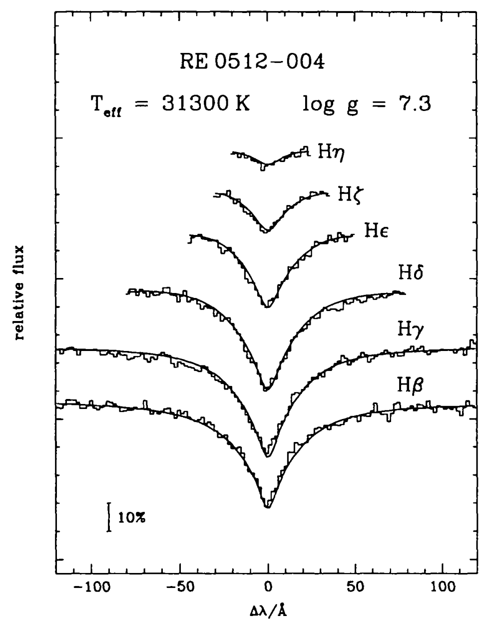

Figure 9: Fit of the DA white dwarf RE 0512-004 (Napiwotzki et al. 1998).

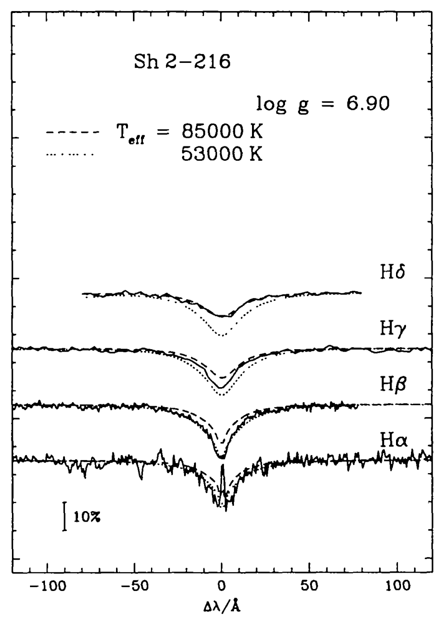

Figure 10: Model spectra calculated for two different temperatures are compared to the Balmer lines of Sh 2-216.

However, this method failed for nearly all hydrogen-rich central stars of old PNe (Napiwotzki 1992; Napiwotzki & Rauch 1994): no consistent fit to the Balmer lines (Hα to Hδ were used) was possible. A strong trend is present: fitting of higher Balmer lines yields higher temperatures. This effect is demonstrated for the spectrum of the white dwarf central star Sh 2-216 in Fig. 10. While a high T_{eff} model (85000 K) fits the Hδ line, only 53000 K are needed to fit the Hα line. This difference causes serious problems for the interpretation of the evolutionary status of white dwarf central stars. The previous investigation carried out by Méndez et al. (1988a) restricted the analysis to one Balmer (Hγ) line only and thus failed to recognize this discrepancy.

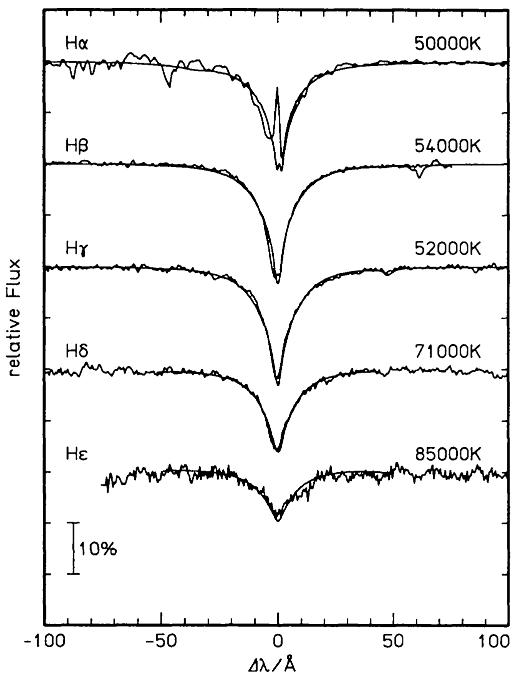

Figure 11: Best fits to the Balmer lines of BD+28°4211 with $\log g = 6.2$ and individually chosen temperatures. The opacity missing in the left wings of Hα and Hβ is related to the He II components which are not well reproduced at these temperatures.

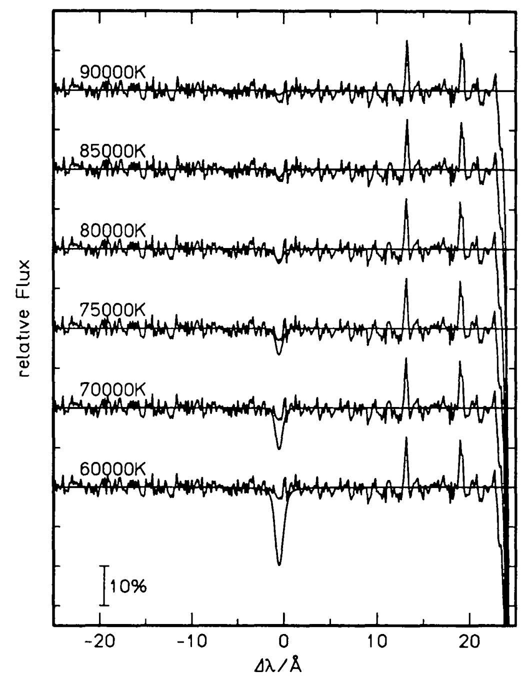

Figure 12: Use of the He I 5876 Å line for the temperature determination of BD+28°4211.

Napiwotzki (1993) argued that the high temperature derived from the highest Balmer line available (Hδ or Hϵ respectively) is closest to the real temperature of the white dwarf central stars. Important evidence came from the analysis of the sdO star BD+28°4211. Its spectral appearance is very similar to central stars of old PNe. Indeed, Zanin & Weinberger (1997) reported recently the detection of extended nebular filaments, which possible belong to a senile PN surrounding BD+28°4211 with an angular diameter of 5°. As can be seen from Fig. 11 the Balmer line problem is of similar strength as it is in the central star of Sh 2-216 (Fig. 10). Since BD+28°4211 is relatively bright ($V = 10.5$) we were able to obtain a high-resolution spectrum and detect the 5876 Å line of neutral helium (Fig. 12). He I lines are very sensitive temperature indicators in this parameter range. We derived a best fit with $T_{\rm eff} = 82000$ K in excellent agreement with the temperature determination from the Hϵ line (85000 K).

Werner (1996a) has demonstrated recently that the inclusion of Stark broadening for C, N, and O lines can have a strong influence on the atmospheric structure of very hot hydrogen-rich stars. It is likely that the effect on the emergent spectrum is pronounced enough to solve the Balmer line problem. An application of his model atmospheres on BD+28°4211 gave good agreement with the observed spectrum, if the parameters of Napiwotzki (1993) were used. Since the computation of a NLTE model grid accounting for the influence of C, N, and O in the above described manner requires very large amounts of computer time, we analysed the CPNe with our atmospheres containing H and He. Since Werner's results show that the temperatures derived from the highest Balmer lines are the most reliable, close to the "real" temperature of the central stars. Since the Balmer line fits result in the same gravity for every Balmer line (within the error limits) we used the following recipe: g was computed from the average of all Balmer lines and $T_{\rm eff}$ is derived from the fit of Hδ (and Hϵ if available). Sample fits are displayed in Fig. 13. A 39 and NGC 6720 are typical examples of white dwarf central stars. Sh 2-174 and HDW 4 are discussed below.

3.3 Results

A $T_{\rm eff}$-g diagram with the results is given in Fig. 14 (from Napiwotzki 1998). The comparison with evolutionary tracks yields a mean mass of $0.58\,M_{\odot}$. This is in good agreement with mass determinations of white dwarfs (cf. Koester et al. 1979, Bergeron et al. 1992). The central stars of old PNe fit nicely into the reported "gap" of the hydrogen-rich sequence. Now a continuous hydrogen-rich sequence from the central star to the white dwarf region is uncovered and therefore evolutionary scenarios claiming a hydrogen-poor stage of all pre-white dwarfs (Fontaine & Wesemael 1987) can be ruled out. Most stars are well explained by the canonical post-AGB evolution (Fig. 14).

However, three stars (HDW 11, Sh 2-174, K 2-2) lie in a region of the HR diagram not crossed by post-AGB tracks (even with very conservative error bars). Two similar objects (EGB 5, PHL 932) were discovered by Méndez et al. (1988a,b). The HR position of some of these stars can be explained by evolution from the extreme horizontal branch (EHB). However, post-EHB evolution is too slow to account for the presence of a PN (cf. Dorman et al. 1993). The position of two stars (HDW 4 and HaWe 5) is (within the error bars) in agreement with post-AGB evolution. However, the time since the AGB was left would amount to more than $3 \cdot 10^6$ years, if standard evolution is assumed. This is at variance with the kinematical ages (< 10000 years) of their nebulae.

An alternative scenario for all five objects is close binary evolution. During the giant stage (on the AGB or RGB) of the central star precursor, it can fill its Roche lobe or even enclose the companion in its atmosphere (common envelope). This causes heavy mass loss and a modification of the stellar evolution. The result is a hot star inside an expanding shell, which can mimic a normal PN (see e.g. the review of Livio 1993). An alternative scenario assumes that the hydrogen-rich envelope was stripped away without a common envelope phase along the (first) red giant branch (Kippenhahn 1967; Iben & Tutukov 1986), which produces a low mass white dwarf with a helium core. The dashed lines are evolutionary sequences for helium white dwarfs calculated by Driebe et al. (1998) based on this scenario. Sh 2-174 is well explained by the $0.414 M_{\odot}$ helium white dwarf track. The model reaches the positon of Sh 2-174 100000 years after it left the RGB. That is in reasonable agreement with the rough estimate of 42000 years for the nebular expansion age given by Tweedy &

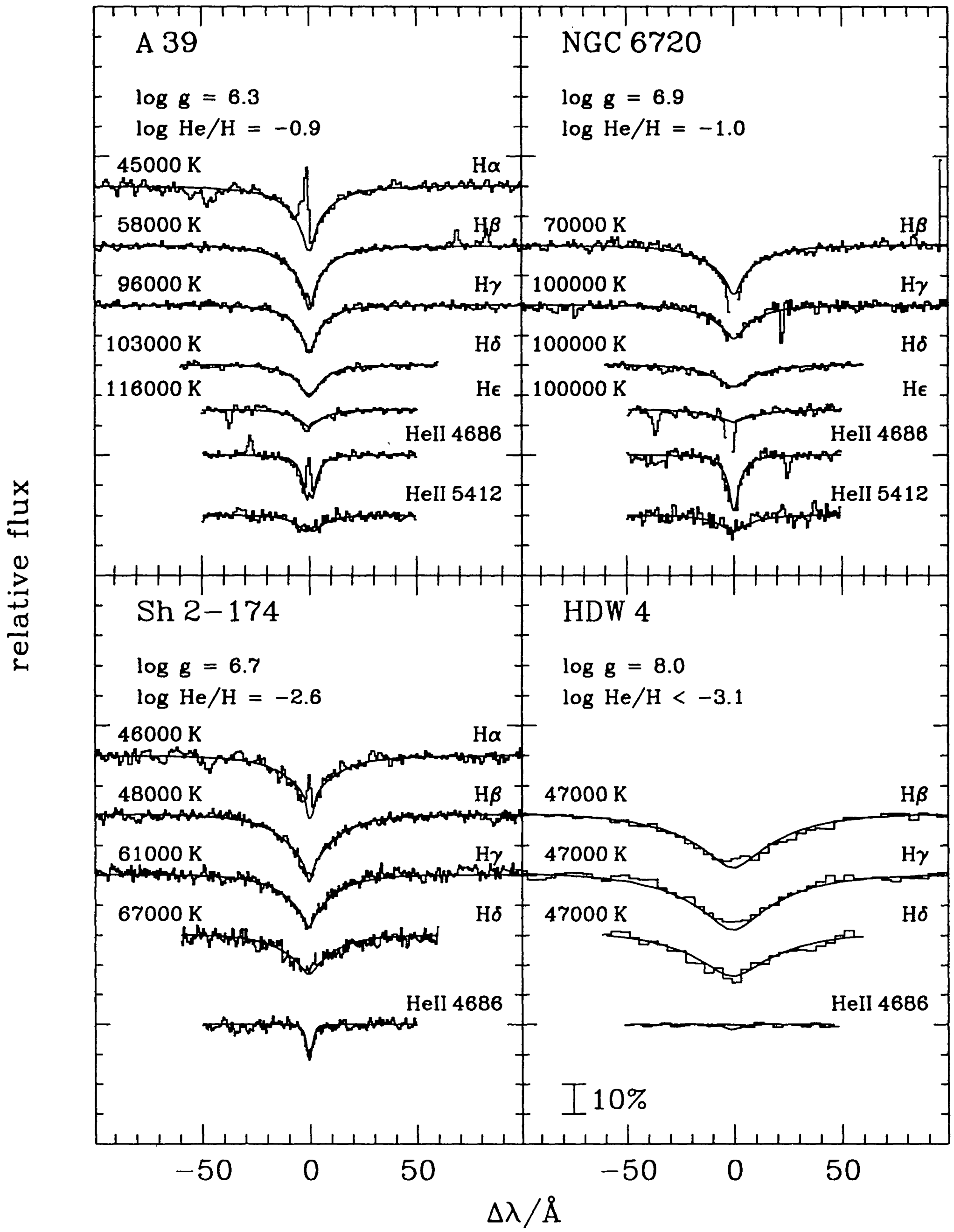

Figure 13: Line fits of selected central stars. Stellar parameters are provided in the plots. For each Balmer line an individually fitted temperature is given.

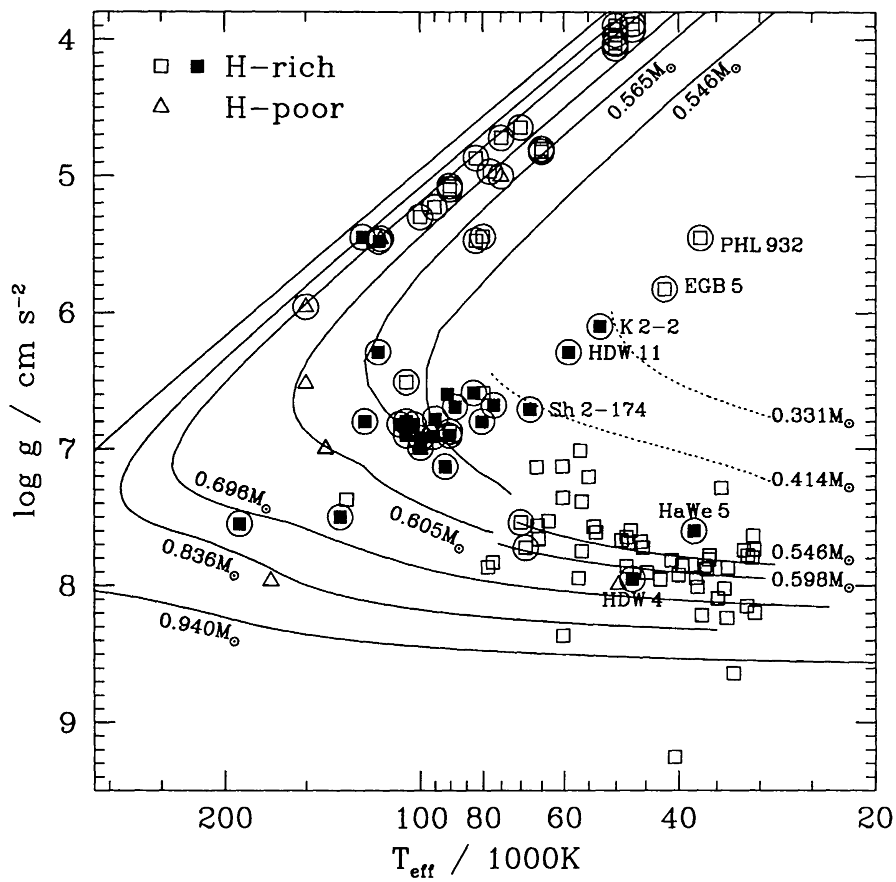

Figure 14: $T_{\rm eff}$-g diagram with the results of the new analyses (Napiwotzki 1998; filled symbols) supplemented by results from literature (Méndez et al. 1981, 1988a,b; Bergeron et al. 1992, 1994; Napiwotzki et al. 1993; Husfeld et al. 1989; Dreizler et al. 1995). Hydrogen-rich objects are marked by squares, hydrogen-poor by triangles. Central stars are encircled. Post-AGB evolutionary tracks were taken from Schönberner (1983), Koester & Schönberner (1986), and Blöcker (1995b). The dashed lines show the helium white dwarf tracks computed by Driebe et al. (1998).

Napiwotzki (1994). It's possible to explain the positions of HDW 11, K 2-2, EGB 5, and PHL 932 with helium white dwarf tracks as well. However, age becomes a serious problem. The post-RGB age of the $0.331 M_\odot$ track at the position of K 2-2 is 700000 years. That is 15 times larger than the PN expansion age estimated by Napiwotzki & Schönberner (44000 years). This discrepancy is even more serious for EGB 5 and PHL 932. Since the post-RGB evolutionary time scales (not the tracks themselves) depend on the adopted mass loss, one may speculate that the age problem may vanish, if the mass loss is varied within reasonable limits. On the other hand it may become necessary to invoke other evolutionary scenarios for some of these stars. Quantitative evolutionary calculations, which may explain the existence of Ha We 5 and HDW 4 and their PNe, are lacking.

Although we have now discussed in detail all central stars, which don't fit into the single star evolutionary scenario, let us emphasize that the majority of our white dwarf central star sample (23 of 28) are well explained by the canonical post-AGB evolution.

4 What does the DB gap tell us?

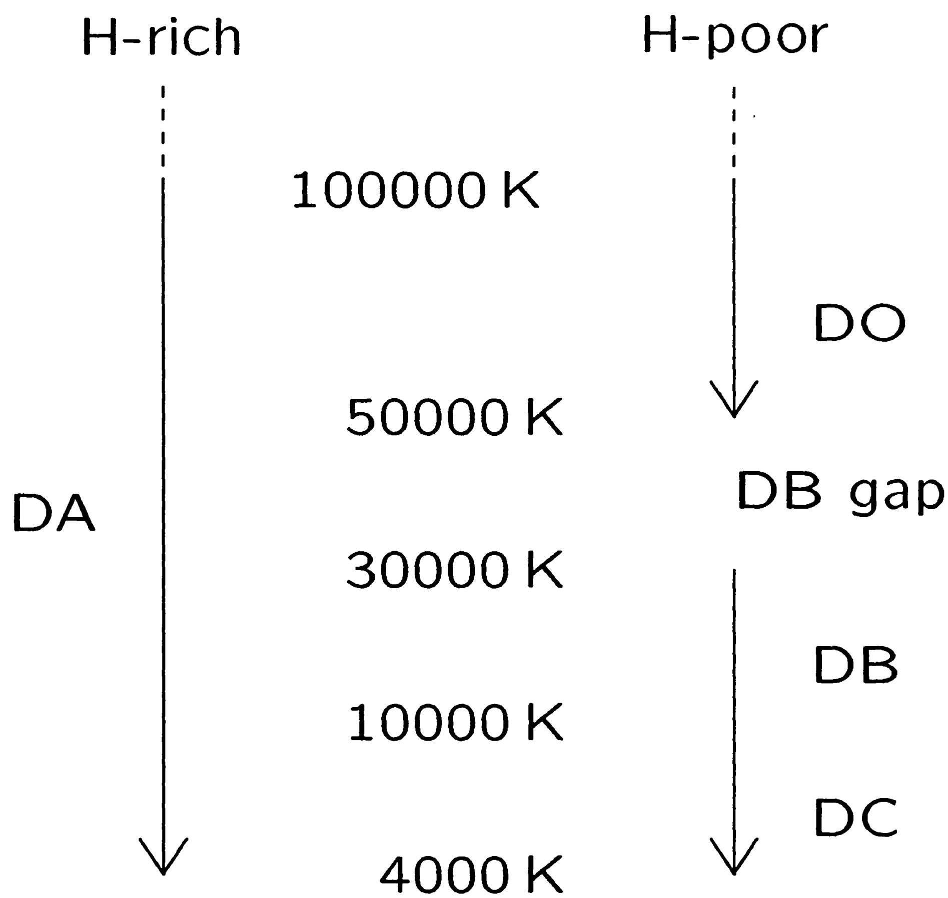

Figure 15: Modified scenario for the spectral evolution of CPNe and white dwarfs with a continuous hydrogen-rich sequence

We have now seen that a continuous hydrogen-rich sequence of CPNe and white dwarfs exists. It is expected that the typical representative of this sequence has a "thick" envelope, as predicted by stellar evolution theory. An accordingly modified version of the scenario presented in Fig. 4 is shown in Fig. 15. However, we still have to explain the existence of the DB gap (that means the absence of helium-rich white dwarfs with $30000 \leq T_{\rm eff} \leq 48000\,$K).

Although the surface abundances of DA and DO/DB white dwarfs are drastically different, the inner structure of both types are virtually identical. Therefore the evolutionary time scales are similar and the ratio of DA to non-DA white dwarfs should stay constant for all temperatures. The DB gap implies that DO white dwarfs manage to mascarede as DA stars and become helium-rich again at the cool end of the DB gap.

The float-up of hydrogen, which remained diluted in deeper layers of the envelope, predicted by the one-channel scenario of Fontaine & Wesemael (1987) provides a natural explanation. Due to gravitational settling the hydrogen would form a very thin surface layer, just opaque enough to hide the helium layer. While we discarded the one-channel scenario, the float-up of hydrogen remains a interesting hypothesis to explain the DB gap. Definite proof and an understanding of what drives the dramatic change of spectral type can be expected, if we find objects in the transition phase. Next we will discuss objects, which are probably just undergoing a change of spectral type.

4.1 Transition from spectral type DO to DA?

The Fontaine & Wesemael (1987) scenario requires that some hydrogen is hidden in the DO envelopes. However, the detection of hydrogen in DO white dwarfs is a difficult task, because all hydrogen lines are blended with much stronger He II lines. In their spectral survey of most known DO white dwarfs Dreizler & Werner (1996) found no evidence for trace hydrogen in any star. However, due to the blending problem typical upper limits on the hydrogen content are as high as 10% or even higher. This is not good enough to constrain possible scenarios. High resolution follow-up observations (Werner 1996b, Dreizler et al. 1998) enabled the detection of hydrogen in one DO (PG 0038+199) out of four. PG 0038+199 is very hot ($T_{\rm eff} = 115000$ K) and Dreizler et al. determined a hydrogen content of 20% by number.

The cool DO white dwarfs are most interesting for a test of the hydrogen float-up hypothesis. From a theoretical point of view one would expect to find stars with relatively high hydrogen content in their atmospheres and from an observational point of view the detection of hydrogen becomes easier for lower temperatures, because the He II components of the blends become weaker while the hydrogen components become stronger. Indeed, Napiwotzki et al. (1995) found evidence for the presence of hydrogen in the atmosphere of the cool DO HD 149499 B. Although it is the brightest DO star known ($V \approx 11.7$), it is very difficult to obtain an optical spectrum because of a bright, close-by main-sequence companion (spectral type K0V). The situation improves in the FUV region, where the white dwarf dominates.

This prompted Napiwotzki et al. (1995) to take an FUV spectrum with the EUV/FUV Berkeley spectrometer of the ORFEUS space experiment (Fig. 16). Temperature and gravity were determined by a simultaneous fit of the He II $2 \rightarrow n$ series in the ORFEUS wavelength range. Every other He II line is blended by a hydrogen Lyman line and the fit improved significantly by adding some hydrogen. The final best fit results shown in Fig. 16 are $T_{\rm eff} = 49500$ K, $\log g = 8.0$ and $n_{\rm H}/n_{\rm He} = 0.22$. The effect of hydrogen on the He II line profiles is demonstrated in Fig. 17. Since the line profile changes are relatively small, the observed spectrum is smoothed for noise reduction and we display a $n_{\rm He}/n_{\rm H} = 1$ model for comparison, instead of the best fitting model, to clarify the effect. The $n_{\rm He}/n_{\rm H} = 60$ model yields a profile which is too broad for the He II 1085 Å line without a hydrogen component and the H+He II 1025 Å profile is too narrow. A better fit can be derived with a higher hydrogen content. This trend is present for all analysed lines, which gives some confidence in this result.

The detection of hydrogen and the closeness to the DB gap makes HD 149499 B a unique object for the understanding of the transition process from DO into DA white dwarfs. The accuracy of the parameter determination of HD 149499 B is somewhat limited by the absence of the He I lines, which are accurate temperature

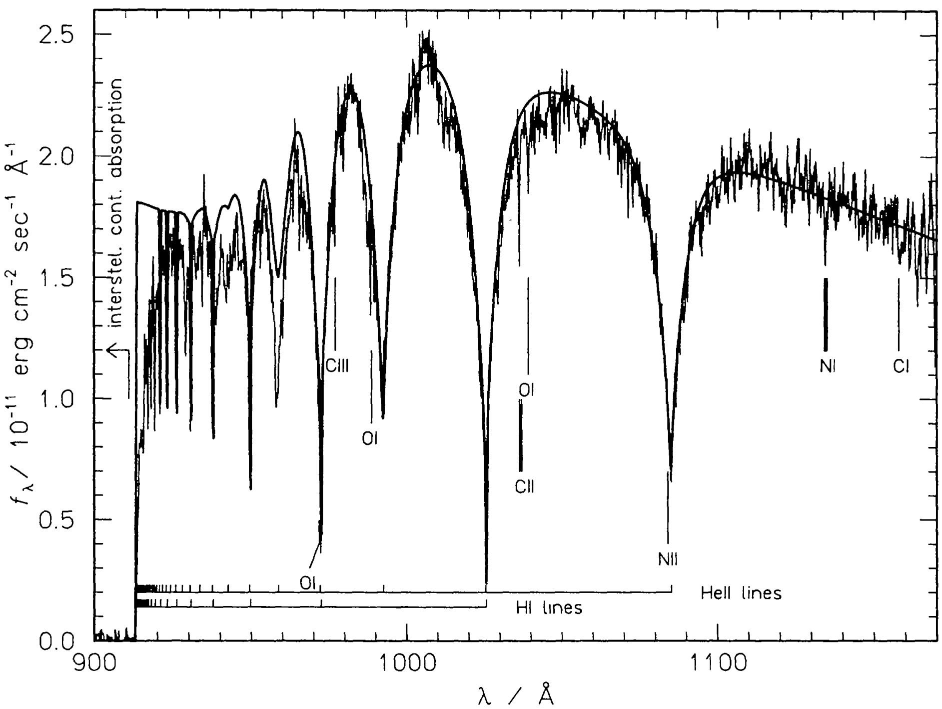

Figure 16: Best fit to the FUV spectrum of HD 149499 B taken with the Berkeley EUV/FUV spectrometer of ORFEUS (Napiwotzki et al. 1995). Important lines are identified. All lines besides the helium and hydrogen lines are of interstellar origin. Due to possible straylight problems in the 924...964 Å region we restricted the fit to $\lambda > 965$ Å.

indicators, in the observed FUV range. Thus Jordan et al. (1997) analyzed an Extreme-UV (EUV) spectrum of HD 149499 B obtained with the EUVE satellite in order to put further constraints on the atmospheric parameters.

The EUV fluxes of stars react sensitively on the stellar temperature, which makes them a good temperature indicator. Since the interstellar hydrogen absorption can be quite high in the EUV range, a simultaneous determination of $T_{\rm eff}$ and interstellar column density is necessary. The effect of changing $T_{\rm eff}$ on the EUV flux of HD 149499 B is demonstrated in Fig. 18. The best agreement is reached for $T_{\rm eff} = 49500$ K with rather small error limits. This is an excellent confirmation of our analysis of the ORFEUS FUV spectrum. Thus, HD 1499499 B is a DO with hydrogen in its atmosphere close to the DB gap.

The most plausible explanation is that HD 149499 B is currently transforming from DO to DA. However, the results presented in Dreizler et al. (1998) indicate that nature might be not as simple as we would like it to be. The only other DO with a hydrogen detection (PG 0038+199) has a rather high temperature (115000 K). Dreizler et al. give upper limits on the hydrogen abundance for six cool DO white dwarfs with $T_{\rm eff} < 60000$ K, which are below the amount detected in HD 149499 B. In particular the upper limit for the coolest white dwarf in their sample (PG 1133+489) is only 1% (compared with the 22% found on the atmosphere of HD 149499 B). Therefore the hydrogen abundance is not a simple function of $T_{\rm eff}$, which is, however, not a direct contradiction to the float-up scenario. The group of DO white dwarfs might simply not be as uniform as it seems at first glance and the

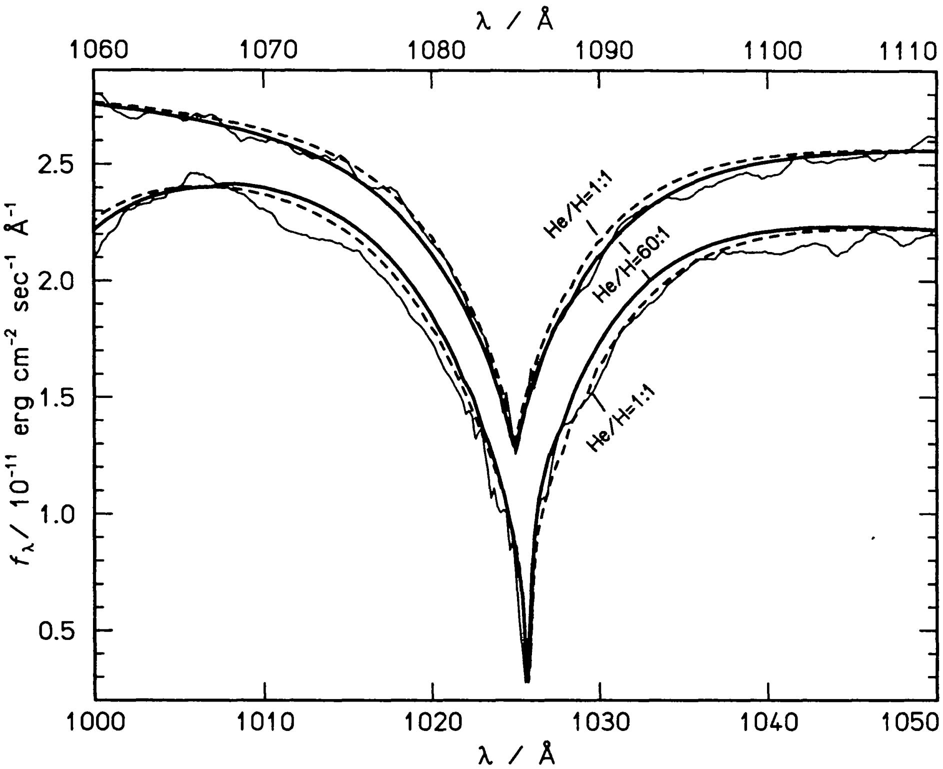

Figure 17: Influence of the hydrogen content on the profiles of the He II 1085 Å line (top) and the H+He II blend at 1025 Å (bottom). We compare with model spectra for $n_{\rm He}/n_{\rm H} = 60$ and $n_{\rm He}/n_{\rm H} = 1$.

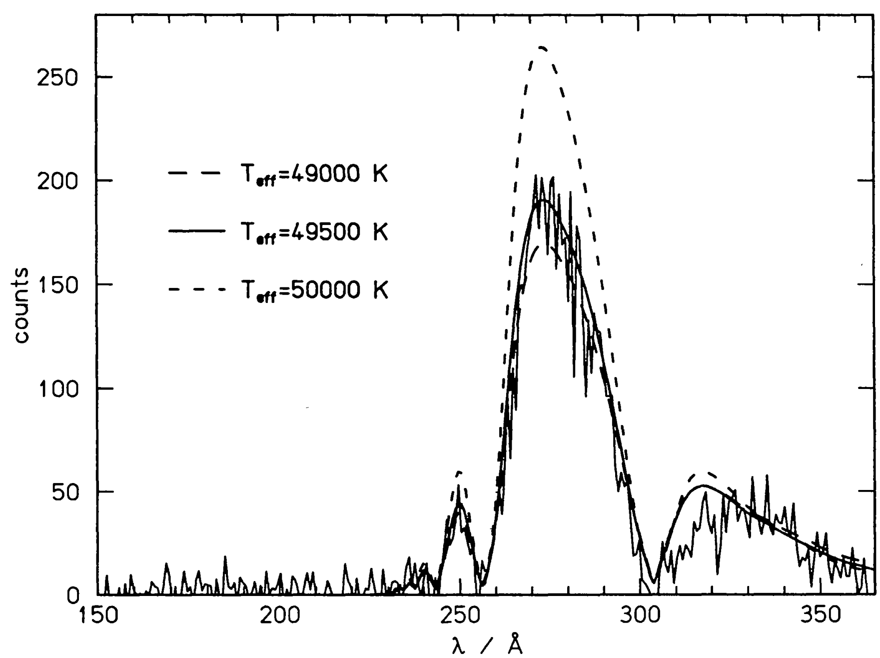

Figure 18: Determination of the temperature of HD 149499 B from the EUVE-MW spectrum (Jordan et al. 1997). The interstellar hydrogen column density is adjusted to match the long wavelength part of the spectrum.

transition time may vary from star to star. Some evidence for important individual variations comes from the spread of C, N, and O abundances found by Dreizler et al. for DO white dwarfs with similar temperatures.

4.2 Transition from spectral type DA back to DB?

DAB stars are hydrogen-rich (basic type DA) white dwarfs, which show at least one line of neutral helium (He I). This is a rare class consisting of only 12 members listed in the new version of the McCook & Sion (1998) catalogue. In Fig. 19 the spectra of the DAB prototype GD 323 (Oke et al. 1984, Liebert et al. 1984) and the recently discovered HS0209+0832 (Jordan et al. 1993) are shown.

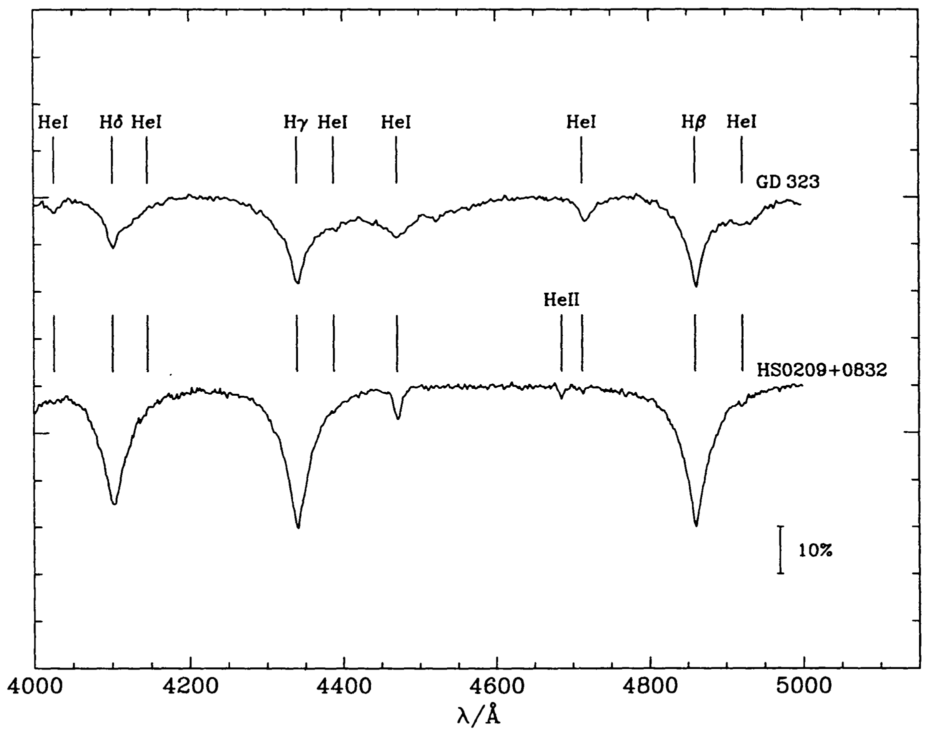

Figure 19: Spectra of the DAB white dwarfs GD 323 and HS0209+832. Important lines are marked.

Most objects have effective temperatures, which place them close to the cool end of the DB gap (≈30000 K). The hot counterparts of the DAB white dwarfs are the DAO white dwarfs. As the calculations of Unglaub & Bues (1998) have shown, it is reasonable to assume that radiation pressure and mass loss are efficient enough to counteract the tendency of gravitational settling in the hot, high gravity atmospheres of DAO stars. However, due to the lower temperature and luminosity radiation pressure as well as mass loss should be much smaller in DAB white dwarfs, far from being able to support detectable traces of helium at the surface. Therefore the mere existence of DAB stars caused some headache. Four explanations were offered in literature:

- a DAB like spectrum can be produced by a composite spectrum of a DA and a DB binary,

- an extremely thin hydrogen layer on top of the helium envelope may produce the Balmer lines, while the helium layer is still visible,
- an interplay between diffusion and convection in stars at the cool end of the DB gap,
- helium rich star spots on the stellar surface.

What is realized in nature? Wesemael et al. (1994) analyzed two DAB stars detected by the Montreal-Cambridge-Tololo survey (MCT 0128-3846, MCT0453-2933). The fit of both, homogeneous and chemically stratified, model atmospheres to the optical line spectrum is poor. Moreover, the resulting stellar temperatures (≈27000 K for both stars) are in serious disagreement with the measured UV and optical energy distribution, Wesemael et al. demonstrated that a good fit can be achieved, if they assume composites of DA and DB spectra. This yields better fits of the optical spectrum and the energy distribution. These stars are no genuine DAB stars, but binaries.

Another type of DAB stars is represented by HS 0209+0832 (Heber et al. 1997) and G 104-27 (Kidder et al. 1992). HS 0209+0832 is best fitted by a homogeneous model atmosphere. The presence of the He II 4686 Å line effectively rules out any combination of DA and DB spectra. The helium lines of HS 0209+0832 and G 104-27 are variable. While Kidder et al. reported only the fading of the helium lines in the case of G 104-27, Heber et al. observed the recovery to the previous strength in HS 0209+0832 (Fig. 20). Both stars can be explained by a helium spot model. A slow rotation of the white dwarf leads to a modulation of the helium lines. Since G 104-27 lies close to the red edge of the DB gap ($T_{\rm eff}$ = 26000 K; Holberg et al. 1990), the interplay of diffusion and convection offers an alternative explanation. Since HS 0209+0832 is hotter ($T_{\rm eff}$ = 36100 K; Heber et al. 1997) the atmosphere is not convective and this scenario can be ruled out.

GD 323 (Fig. 19) is another DAB close to the red edge of the DB gap ($T_{\rm eff}$ = 28750 K; Koester et al. 1994). Koester et al. considered homogeneous and stratified atmospheres. helium spots and the possibility that GD 323 could be a DA + DB binary. Neither model perfectly reproduced the observations, but the authors regarded the stratified model as the best match. Although we have not yet completely understood the physics of GD 323, it is a good candidate for an object just transforming from spectral type DA to DB.

5 Conclusions

Our knowledge of the central star – white dwarf transition region has dramatically increased during the last years. The survey of central stars of old PNe by Napiwotzki & Schönberner (1995) resulted in 38 classifications, multiplying the number of previously known stars. The ratio of hydrogen-rich to -poor star amounts to 4:1. This is consistent with the findings of Liebert (1986) and Méndez (1991). The analysis of the H-rich CPNe is hampered by the Balmer line problem: for most stars a consistent fit to all Balmer lines was not possible. Generally the highest Balmer line yields the highest temperature. It was shown by Werner (1996a) that the problem can be solved by the inclusion of light metals in the atmospheric

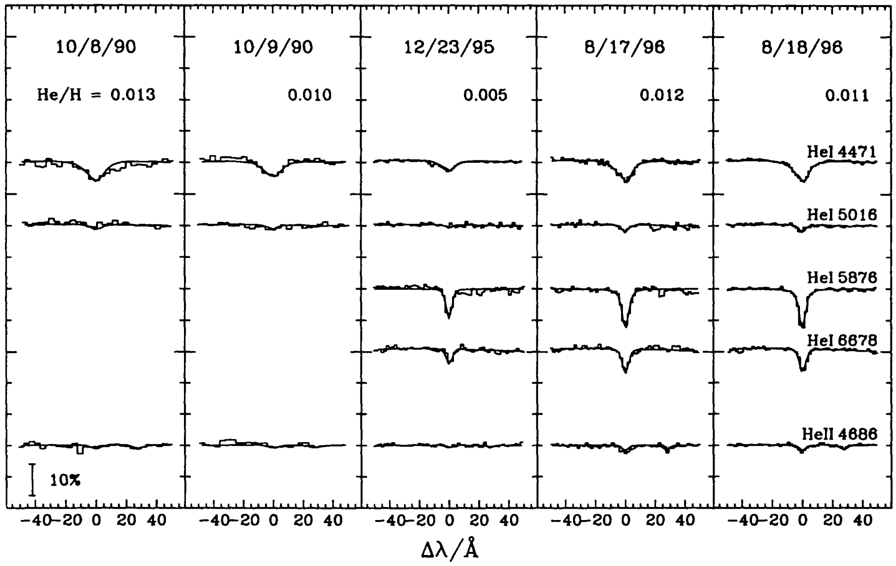

Figure 20: Variations of the helium lines of HS 0209+0832 at five epochs (Heber et al. 1997). The best fits are shown and the resulting helium abundance is given. Both spectra taken in 1990 are of lower resolution than the spectra taken in 1995 and 1996 (7 Å and 3.6 Å respectively).

computations in an appropriate way. Due to practical considerations we relied on atmospheres composed only of H and He and used Hδ as temperature indicator as suggested by the more sophisticated models of Werner (1996a).

The investigation revealed that the reported gap in the hydrogen-rich evolutionary sequence is not real. Taking into account the hot white dwarfs analyzed by Bergeron et al. (1994) we have now a continuous H-rich sequence from the CPNe to the white dwarfs. The observational findings are well explained by a two channel scenario with a H-rich and a H-poor sequence. It is not longer necessary to invoke an one channel scenario (Fontaine & Wesemael 1987) to explain the spectral evolution of pre-white dwarfs. Five central stars are not in agreement with standard post-AGB evolution. They are possibly the result of close binary evolution.

Although we closed the "central star gap" the DB-gap in the helium-rich white dwarf sequence still needs explanation. In this case the very thin layer scenario proposed by Fontaine & Wesemael (1987) is the best existing scenario. It predicts that stars change their spectral type from DO to DA at the hot end of the DB gap and back from DA to DB at the cool end. A candidate for the DO to DA transition is the cool DO HD 149499 B, which has a hydrogen fraction of 20% in its atmosphere. Candidates for the DA to DB transition are the DAB stars. We show that this class is rather inhomogeneous and consists of spectroscopic DA+DB binaries (MCT 0128-3846, MCT0453-2933), stars showing helium line variability (HS 0209+0832, G104-27) best explained by helium star spots, and only one star (GD 323) which is best explained by a very thin hydrogen layer.

Although a consistent picture of the spectral evolution of central stars of PNe and white dwarfs is emerging now, a closer look reveals that several details cannot be explained by one simple scenario. White dwarfs show more individuality than we like them to. Great progress was achieved during the last decade, but we need more information, before we can hope to derive a good understanding of the spectral evolution during the final stages of low and intermediate mass stars.

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