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The age and interior rotation of stars from asteroseismology

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We provide a status report on the determination of stellar ages from asteroseismology for stars of various masses and evolutionary stages. The ability to deduce the ages of stars with a relative precision of typically 10 to 20 % is a unique opportunity for stellar evolution and also of great value for both galactic and exoplanet studies. Further, a major uncalibrated ingredient that makes stellar evolution models uncertain, is the stellar interior rotation frequency $\Omega(r)$ and its evolution during stellar life. We summarize the recent achievements in the derivation of $\Omega(r)$ for different types stars, offering stringent observational constraints on theoretical models. Core-to-envelope rotation rates during the red-giant stage are far lower than theoretical predictions, pointing towards the need to include new physical ingredients that allow strong and efficient coupling between the core and the envelope in the models of low-mass stars in the evolutionary phase prior to core helium burning. Stars are subject to efficient mixing phenomena, even at low rotation rates. Young massive stars with seismically determined interior rotation frequency reveal low core-to-envelope rotation values.

1 Asteroseismology: the new route for stellar physics

Contemporary stellar structure and evolution theory still has several open questions, the answers having vast implications for exoplanetary, supernova, and (extra)galactic science. One major uncalibrated quantity is the rotation frequency in the stellar interior throughout stellar life. Also the level of chemical mixing inside stars is hard to judge from surface abundance measurements. While models of stars not too different from the Sun in terms of mass, rotation, and evolutionary stage are well scalable from the solar model, this is far less so for stars with appreciably different properties such as high mass, rapid rotation, strong wind, evolved status, etc.

A recent driver to improve stellar physics is the availability of long-duration high-cadence quasi-uninterrupted white-light space photometry with a precision of μmag assembled for thousands of stars by the European CoRoT (operational from 2006 to 2012, Auvergne et al. 2009) and the NASA *Kepler* (operational from 2009 to 2013, Koch et al. 2010) satellites. This offered the opportunity to confront stellar models with *asteroseismic* data. This is done by computing the predicted spectrum of normal oscillation modes from theoretical models, by considering small perturbations to the equations of stellar structure, usually in the approximation of a spherically symmetric star. Normal modes are either *pressure* (p-)modes or *gravity* (g-)modes, depending on whether the pressure force, respectively buoyancy, is the dominant restoring force. Pressure modes probe the stellar envelope while g-modes tune the inner part of star. Such seismic diagnostics are far more suitable to probe stellar interiors compared to measurements of surface quantities. Figure 1 shows part of a typical *Kepler* light curve for a newly discovered g-mode pulsator, as well as the frequency spectrum based on the entire light curve in the range of maximum amplitude. An extensive description of the method of asteroseismology is available in Aerts et al. (2010).

Seismic modelling consist of searching for a theoretical stellar model whose frequency spectrum matches the observed one in the bottom panel of Fig. 1 to within the measurement errors. A basic scheme of how forward seismic modelling works is available in Aerts (2013, Sect. 1 and Fig. 1) and is hence not repeated here. For each of the detected pulsation frequencies, $\omega_{n\ell m}$, identification of the radial order n and spherical wavenumbers, (ℓ, m) representing the nodes of the eigenmode, is necessary in order to perform meaningful forward modelling. Once mode identification has been achieved, the seismic modelling is usually based on the frequency values of zonal ($m = 0$) modes, which represent standing waves that are not affected by the stellar rotation in the case of a spherically symmetric star. The modes with $m \neq 0$ represent waves travelling in the direction of rotation (prograde modes) or opposite to it (retrograde modes) and can in principle be used to derive the internal rotation frequency $\Omega(r)$ after a good seismic model based on the zonal modes has been achieved. We come back to this opportunity in Sect. 3.

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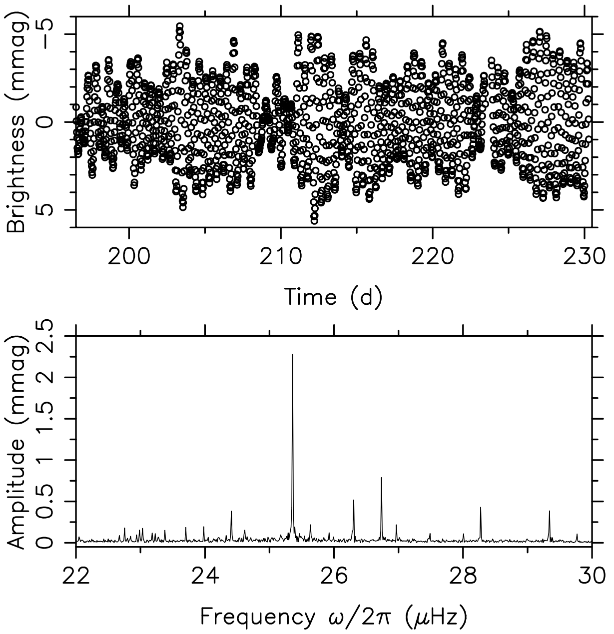

Fig. 1 *Top*: excerpt from the *Kepler* light curve of the g-mode pulsator KIC 9210943; *bottom*: frequency spectrum in the range of the dominant modes based on the full light curve spanning 1470 d.

In the limits of high- or low-frequencies, identification of (n, ℓ, m) can often be achieved from frequency patterns. The *large frequency separation* of p-modes of high radial order n, e.g., is the difference in frequency of modes with the same degree but of consecutive radial order. This separation equals the inverse of twice the sound travel time between the stellar surface and centre and offers a high-precision measurement of the mean density of the star. *Stochastically excited modes* are triggered by the convective envelope of stars and these modes appear as Lorentzian peaks in Fourier space. They fulfill so-called *scaling relations* based on the large frequency separation and the *frequency of maximum power*, allowing to estimate the stellar mass and radius when the effective temperature is known (Kjeldsen & Bedding 1995). The validity of these relations was already confirmed for solar-type stars with pulsations detected in ground-based radial-velocity data (e.g., Bouchy & Carrier 2001; Bedding et al. 2004; Kjeldsen et al. 2005) and have now seen widespread application to extensive ensembles of unevolved solar-type stars (e.g., Chaplin et al. 2014) and red giants (e.g., Hekker et al. 2011; Huber et al. 2011; Kallinger et al. 2012; Stello et al. 2013).

For g-modes, a natural diagnostic to consider in forward modelling is the *period spacing* between high-order low-degree modes of consecutive radial order. This period spacing is connected with the detailed properties of the buoyancy frequency inside the star and probes the deep interior. As a star grows old, it builds up chemical composition gradients due to its nuclear burning stages. Hence, *trapping* of g-modes occurs in the deep interior and this gives rise to deviations from uniform period spacings (e.g., Brassard et al. 1992; Miglio et al. 2008).

A major discovery based on *Kepler* light curves is the occurrence of dipole mixed modes in red giants (Beck et al. 2011; Bedding et al. 2011). These were predicted theoretically by Dupret et al. (2009) and have a p-mode character in the star's envelope while g-mode properties in the core region. They allow the combined use of the large frequency separation and the period spacing as an important diagnostic to deduce if the red giant is in the hydrogen shell-burning phase or rather in the core helium-burning phase (Bedding et al. 2011). Such a distinction cannot be made without seismic data because the surface properties of the red giants are alike.

Below, two important "deliverables" of asteroseismology for astrophysics are emphasized. It concerns the age and interior rotation profile of stars. Asteroseismology can offer these quantities with a precision that no other method can. By choosing this focus, this paper by no means gives an overview of all the results that have been achieved in asteroseismology lately. Moreover, the text is heavily biased towards modelling results. These rely on the outcome of asteroseismic data analyses. Hence our text does not bring justice to the hundreds of instrumental and observational studies upon which the seismic inferences are based. Extensive earlier reviews of asteroseismology in the space era are available in, e.g., Michel et al. (2008), Gilliland et al. (2010), Christensen-Dalsgaard & Thompson (2011), and Chaplin & Miglio (2011).

2 Stellar ages

Stellar ages are very hard to determine with high precision, while this quantity is of vast importance in astrophysics. The formation and evolution of exoplanet host stars, e.g., directs the formation and evolution of their planets. On the other hand, galactic archeology and nearby cosmology require to identify the oldest well-accessible stellar populations in various regions of the Milky Way and in neighbouring galaxies. This can now be tackled seismically from red giant studies.

While the recent applications of *gyrochronology* of rotationally modulated active stars is highly successful (e.g., Epstein & Pinsonneault 2014; Meibom et al. 2015), this method of aging is limited to stars in a very narrow low-mass range, typically between 0.8 and 1.4 $M_\odot$. Asteroseismic aging can be done for all mass ranges, whenever several non-radial pulsation mode frequencies can be detected and identified. Garciá et al. (2015) bridged gyrochronology and asteroseismic aging for lower-mass stars and found compatible results, thus calibrating the gyrochronology relation seismically. However, they stressed that the age-rotation-activity relations seem quite different for the hotter dwarfs and subgiants. For this reason, and because it works for all masses, seismic aging is a major asset.

2.1 Aging of low-mass solar-like pulsators

2.1.1 Core hydrogen-burning stage

In contrast to the mass and radius, estimation of the stellar age cannot be achieved model-independently from the scaling relations based on the large frequency separation and the frequency of maximum power of solar-like pulsators. It requires the additional measurement of the so-called *small frequency separation*, which is the frequency difference between adjacent radial and quadrupole zonal modes differing by 1 in radial order. This seismic quantity is connected with the sound speed gradient inside the star and hence tunes the regions where nuclear burning has taken place in the past and/or is occurring presently. Coupled to the evolutionary track for the seismic mass deduced from the scaling relations, the small frequency separation delivers the stellar age, *for a particular metallicity and for the chosen input physics of the stellar models*. This method was applied to a large sample of more than 500 solar-like core hydrogen-burning stars and provided relative age precisions of 10 to 15 % (Chaplin et al. 2014).

Detailed stellar modelling of case studies taking into account the frequencies themselves rather than just a few average quantities, offers a much better way to derive the age of low-mass stars. This was demonstrated by Lebreton & Goupil (2014), who presented an extensive study of the exoplanet host star HD 52265 based on four months of CoRoT data. In their study, they quantified the impact of various assumptions for the input physics and free parameters (such as the mixing length), and their uncertainties, used in the stellar models on the derived age. This allowed them to obtain relative precisions on the age, mass, and radius that are smaller than those obtained from the large and small frequency separations alone, although the frequency precision for their case study was limited. Metcalfe et al. (2014) performed a similar but far more extensive study of 42 solar-like stars based on nine months of *Kepler* data. They found that forward modelling based on the individual frequencies typically doubles the precision of the asteroseismic radius, mass, and age compared to estimates based on the scaling relations or on the large and small frequency separations. The full four-year *Kepler* light curves have yet to be explored in terms of detailed forward modelling based on the individual frequencies.

Lebreton & Goupil (2014) stressed the tight correlation between the initial helium abundance and mass of the star. This was taken up by Verma et al. (2014) in the case of the solar analogs 16 Cyg A and B, monitored during 2.5 yr by the *Kepler* mission. These data allowed the authors to determine the current helium abundances of these stars to lie between 0.231 and 0.251 for 16 Cyg A and between 0.218 and 0.266 for 16 Cyg B. This resulted from detailed modelling of the pulsation frequencies, keeping in mind that the helium ionization zone leaves particular signatures on the oscillation frequencies that can be measured from frequency values of sufficiently high precision, as offered by the 2.5 yr time base of the *Kepler* data. These signatures are connected with so-called *acoustic glitches*. These are due to regions in the stellar interior where the sound speed experiences an abrupt variation due to a local change in the stratification. Acoustic glitches occur at the boundaries between radiative and convective regions, as well as in the partial ionisation layers, especially those of hydrogen and helium. Such a glitch introduces an oscillatory component in the pulsation frequencies of the star.

Mazumdar et al. (2014) exploited the measured acoustic glitches of 19 solar-type stars based on nine months of *Kepler* data, along with the large and small frequency separations to locate the base of the convective envelope and the position of the second helium ionisation zone. They found good agreement with state-of-the-art models for these 19 stars. Future applications of actual forward modelling based on frequency fitting may offer the potential to constrain the metallicity and chemical mixture of solar-like stars, by exploiting the full *Kepler* data sets.

2.1.2 Evolved stages

While the small frequency separation is an appropriate age indicator for the core hydrogen burning phase, this is no longer the case for the sub-giant phase. Indeed, subgiants have a rather inert helium core that hardly undergoes evolutionary changes. On the other hand, the stellar core starts shrinking when the star climbs up the red giant branch. The small frequency separation can then again be used to tune the age in that stage, given that it is sensitive to the consequences of the hydrogen shell burning (e.g., White et al. 2011).

During the red-giant branch evolution, the small frequency separation changes gradually until helium ignition through the helium flash occurs. After the flash, when core helium burning occurs in equilibrium after several subflashes, the small frequency separation is typically some 10 % larger than the one on the red-giant branch (e.g., Corsaro et al. 2012; Kallinger et al. 2012). The theory of stellar evolution during core helium burning on the horizontal branch is not yet well enough understood to interpret changes in the small frequency separation in terms of the stellar age alone. Indeed, various physical phenomena such as core overshooting, atomic diffusion, turbulent and/or diffusive mixing, mass loss, and core rotation are active together, while they are yet poorly understood and remain uncalibrated. Each of them has an effect on the small frequency separation. For this reason, Kallinger et al. (2012) suggested to use the detailed properties of the three dominant radial modes in red giant frequency spectra, including their phase behaviour, as an appropriate age proxy that can also be applied to ground-based spectroscopic seismic data of bright pulsating red giants. Nevertheless, as stressed by Mosser et al. (2014) and following Bedding et al. (2011), a measurement of the period spacing of dipole mixed modes along with the large frequency separation of p-modes offers the best way to derive the evolutionary stage of low-mass stars, all the way from the sub-giant phase to the end of the core helium burning.

Another type of core helium burning stars for which forward seismic modelling has been achieved are the subdwarf B (sdB) stars. These objects are situated at the blue end of the horizontal branch and are responsible for the observed UV-upturn of early-type galaxies. It remains unclear how the sdB stars have lost their hydrogen envelope during the red giant branch phase, but binarity is held responsible for it, given that half of the sdB stars reside in close binaries with a white dwarf or low-mass core hydrogen-burning companion (Heber 2009). While red giant pulsation are excited by the convection in the outer envelope, sdB stars have no such envelope but may experience pulsation modes excited in the partial ionisation layers of iron-like elements. Seismic modelling of sdB stars is a major asset to tune the physics of core helium burning stars, but was so far limited to single-star models starting from the onset of core helium burning, irrespective of the previous evolutionary history. Age estimates are then found with respect to the zero-age horizontal branch (ZAHB). The frequency spectra of sdB pulsators do not always permit to identify the mode numbers (n, ℓ, m). In that case, these are estimated along with the modelling, by considering all modes of low degree to fit the frequencies. While this may seem arbitrary, this procedure was demonstrated to be fully appropriate from the binary versus asteroseismic modelling of the pulsating reflection eclipsing binary sdB pulsator PG 1336–018 (Vučković et al. 2007; Van Grootel et al. 2013).

Van Grootel et al. (2010a) performed the first detailed forward modelling of a g-mode sdB pulsator, KPD 0629 –0016, observed with the CoRoT mission during 21 d. They found a total mass of $M_{\rm tot} = 0.471 \pm 0.002\,{\rm M}_\odot$, a hydrogen envelope mass of $\log{(M_{\rm env}/M_{\rm tot})} = -2.42 \pm 0.07$, and an age of 42.6±1.0 Myr since the ZAHB. These results differ from those for the younger sdB pulsator KPD 1943+4058 observed with the *Kepler* mission in terms of total mass, but are similar for the hydrogen envelope: $M_{\rm tot} = 0.496 \pm 0.002\,{\rm M}_\odot$, $\log{(M_{\rm env}/M_{\rm tot})} = -2.55 \pm 0.07$, for an age of 18.4±1.0 Myr since the ZAHB (Van Grootel et al. 2010b). Charpinet et al. (2011) modelled a third sdB pulsator, KIC 2697388, but could not pinpoint a unique value for the total and envelope masses, because of a degeneracy in two families of solutions of equal quality in terms of seismic modelling. Nevertheless, they derived an upper age limit of ~ 55 Myr since the ZAHB and came to the conclusion that the *seismically modelled sdB pulsators hint towards the need of extra mixing since the ZAHB*, caused by e.g. core overshooting and/or differential rotation, compared to the models based on the Schwarzschild criterion of convection to define the inner convective core. As we discuss below, this conclusion also holds for massive stars with a convective core during the core hydrogen burning.

2.2 Aging of massive pulsators

Not only sdB stars, but numerous classes of stars, in various evolutionary phases, experience so-called self-driven non-radial pulsation modes excited by a heat mechanism, usually due to an opacity bump occurring in partial ionisation layers – see, e.g., Aerts et al. (2010), Chap. 2, for an overview of all those classes and their pulsational properties. The highest-mass star for which such heat-driven non-radial pulsations have been modelled seismically concerns the slowly rotating O9V star HD 46202, leading to mass of 24±0.8 $\mathrm{M}_\odot$, a core overshooting value of 0.10±0.5 local pressure scale heights, and an age of 4.3±0.5 Myr (Briquet et al. 2011). Unlike stochastically-excited solar-like pulsations, the heat-driven modes are not damped and so their frequencies give rise to delta peaks in Fourier space. This makes it easier to determine them with high precision compared to the Lorentzian frequency peaks caused by the continuously damped and re-excited stochastic solar-like pulsations. On the other hand, heat-driven modes do not necessarily occur in a frequency regime that gives rise to frequency separations or period spacings and lack of unambiguous identification of (n, ℓ, m) corresponding with the detected mode frequencies may impose a serious limitation, preventing seismic modelling.

A new way forward in seismic modelling and aging for intermediate- and high-mass stars in core hydrogen burning, based on period spacings due to high-order gravity modes, was initiated by Degroote et al. (2010). They did not only detect the period spacing of 9450 s due to dipole modes in the B3V Slowly Pulsating B (SPB) star HD 50230 of $\sim 7\,\mathrm{M}_\odot$ observed with CoRoT, but they also derived periodic deviations from the constant spacing with a periodicity of 2450 s and an amplitude of some 240 s, decreasing with increasing mode period. This is clearly the signature of *a smooth gradient of chemical composition due to diffusive mixing* inside the star, because instantaneous near-core mixing would leave much sharper features in the period spacing (Miglio et al. 2008), given that forward modelling led to the conclusion that the star has already consumed about 60 % of its initial hydrogen. An absolute age could not be derived for this star, due to too poor constraints on the core overshoot value for which only a lower limit of 0.2 local pressure scale heights could be concluded.

Detections of period spacings have recently also been announced for F-type γ Dor pulsators, both in the case of slow rotators with average spacings of a few thousand seconds (Bedding et al. 2014) and for modest to fast rotators whose period spacings are affected strongly by rotation and have values of only a few hundred seconds (Van Reeth et al. 2015a,b – see Fig. 2 showing one of the sample stars). Such decreased values for the spacings are a clear signature of the effect of rotation on the frequency patterns, as theoretically expected (Bouabid et al. 2013). Moreover, the smooth downward and upward patterns without deep dips as observed in Fig. 2 are a signature of diffusive mixing inside the star, inhibiting strong mode trapping, as outlined

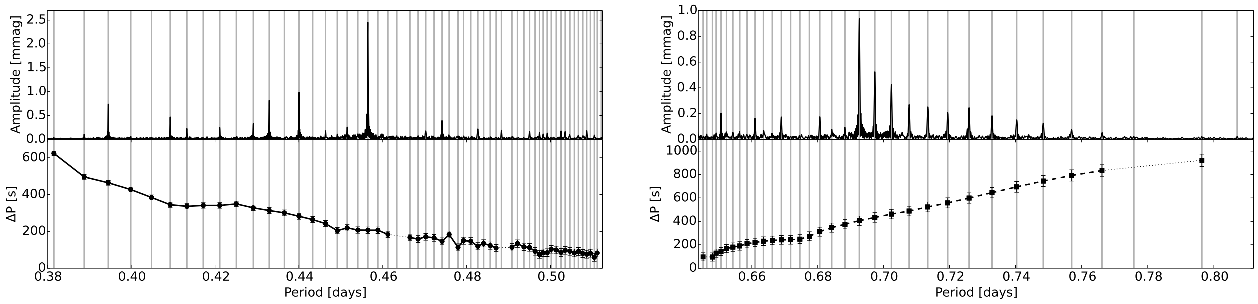

Fig. 2 Period spacings of prograde (*left*) and retrograde (*right*) dipole modes in a *Kepler* F-type pulsator from the sample by Tkachenko et al. (2013). Significant frequencies belonging to the same spacing pattern are indicated with vertical grey lines. Figure reproduced from Van Reeth et al. (2015b).

by Bouabid et al. (2013). While the sample of γ Dor stars by Tkachenko et al. (2013) is still subject of forward seismic modelling based on the observational findings in Van Reeth et al. (2015b), it is already clear that the detection of prograde and/or retrograde dipole period spacings as it has been achieved for 45 γ Dor stars of the sample, one of which shown in Fig. 2, will offer an excellent opportunity to model the interior rotation, chemical mixing, and aging of stars in the mass range of 1.5 to 2.5 $M_\odot$ in the near future.

To conclude this section, we stress that mixing inside stars, particularly when it occurs in the near-core regions of stars with a convective core, impacts directly on the age estimation. Precise aging thus requires a thorough understanding of the relevant mixing processes for representative samples of stellar populations. We have not yet reached that stage for massive core hydrogen burning stars, nor for the core helium burning stars. The available results on seismically tuned mixing concern too few stars in these categories so far. However, asteroseismology is well on its way to achieve appropriate calibrations for mixing processes that are needed to bring the observational seismic data into agreement with theoretical models.

3 Interior rotation

Application of asteroseismology to stars of different mass and rotation is the best route to make progress in the improvement of stellar physics. Indeed, *a key ingredient that remained uncalibrated in stellar evolution theory until recently is the interior rotation frequency $\Omega(r)$ during stellar life.* The angular momentum that stars have at birth, after the completion of the various stages of their formation process, is essentially unknown. The poorly understood star–to–disk interaction and wind and accretion properties of protostars during the contraction phase imply that we cannot quantify the rotation profile $\Omega(r)$ at birth from first principles. Further, the redistribution of angular momentum during the life of the star is critical for its evolution, yet hard to evaluate in practice. Asteroseismology is the best way to deliver constraints on $\Omega(r)$. While this was already achieved from long-term monitoring of a few unevolved p-mode pulsators among massive B-type stars from ground-based data (Aerts et al. 2003; Pamyatnykh et al. 2004; Briquet et al. 2007), leading to estimates of the core-to-envelope rotation $\Omega_{\rm core}/\Omega_{\rm env}$ between 1 and 4, major progress was made recently on the basis of *Kepler* light curves.

While rotation affects the period spacings of g-modes as illustrated in Fig. 2, its clearest signature in any pulsation spectrum is the occurrence of rotationally split multiplets. The frequency of each mode of degree ℓ gets split into $2\ell + 1$ multiplet components due to rotation. Depending on the inclination angle between the rotation axis of the star and the line-of-sight, and on the amplitudes with which the individual multiplet components get excited, all or only a few of the multiplet components can be detected. To achieve this, the time series must be sufficiently long to have enough resolving power for the frequency multiplet detections in the case of slow rotators. This was achieved for 19 g-mode triplets of the SPB star KIC 10526294, leading to a rotation period of 188 d (Pápics et al. 2014) and hints of the presence of counter-rotation in its envelope (Triana et al. 2015), from four years of *Kepler* monitoring. Frequency multiplets of low-frequency g-modes easily get merged for intermediate to fast rotators or they may even lead to power in different frequency regimes for prograde versus retrograde modes, as is the case for the γ Dor star in Fig. 2 whose $v \sin i = 72 \pm 8\ \mathrm{km\,s^{-1}}$ (Tkachenko et al. 2013).

A major asset to deduce rotational information for young unevolved intermediate mass stars are *hybrid pulsators*. These are stars that undergo both p-mode and g-mode pulsations, delivering seismic probing power throughout the entire star. Figure 3 shows the frequency spectra of six such hybrid F-type pulsators observed with *Kepler* and exhibiting pulsational signal in a broad frequency range spanning some 250 μHz. A few hybrid pulsators have recently allowed to estimate $\Omega_{\rm core}$ and $\Omega_{\rm env}$. This research was pioneered by Kurtz et al. (2014) and Saio et al. (2015), who discovered rotationally split triplets and quintuplets in two F-type hybrids and deduced average rotation periods of about 100 and 65 d for them, one star having a slightly faster envelope than core rotation (KIC 11145123) while the opposite is the case for the other (KIC 9244992). These $\Omega_{\rm core}/\Omega_{\rm env}$ are essentially model-independent whenever p- and g-mode multiplets are detected.

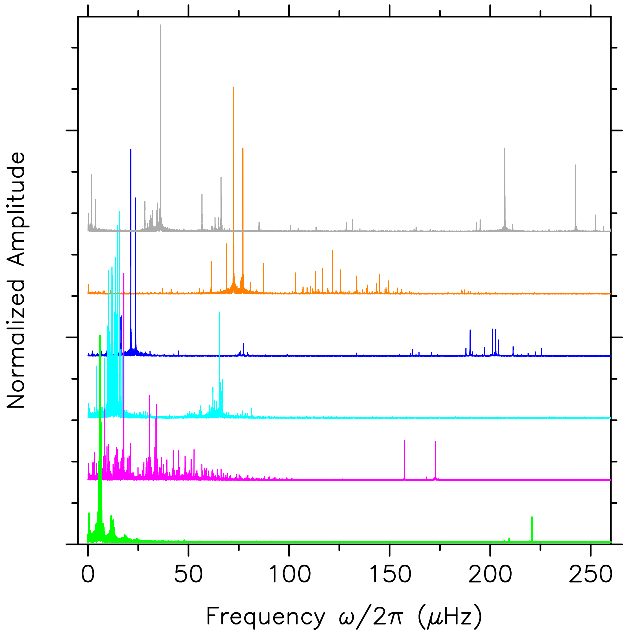

Fig. 3 Pulsation spectra of six intermediate-mass F-type hybrid pulsators in the core hydrogen burning stage observed with the *Kepler* satellite. From *top to bottom*: KIC 4749989, KIC 1849235, KIC 8645874, KIC 9751996, KIC 6468146, KIC 6367159. The dimensionless amplitudes are the measured amplitudes divided by the amplitude of the strongest mode, shifted along the y-axis for visibility purpose. For each of these stars, hundreds of significant frequencies were detected in the shown frequency range.

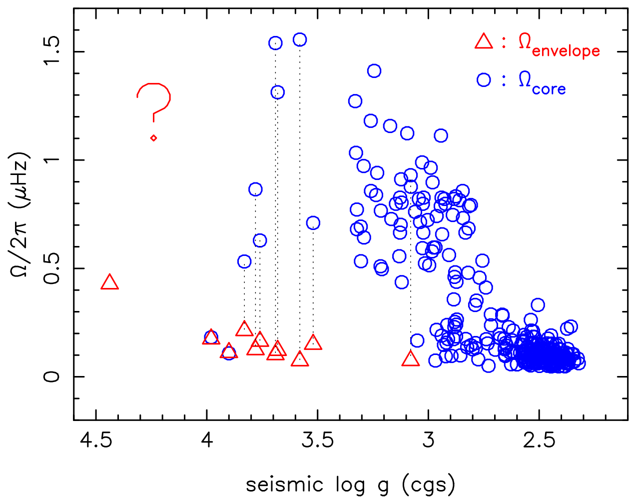

Fig. 4 Core rotation frequency of stars with mass between 0.7 and 3 $M_\odot$ in terms of their seismically determined surface gravity ($\log\, g$) as a rough proxy of their evolutionary stage. The error on the frequencies is smaller than the symbol size, while 0.05 is a typical uncertainty for the seismic $\log\, g$ values. The leftmost triangle represents the envelope rotation of the Sun, averaged over latitude; $\Omega_{\rm core}$ is not available for the Sun. Stars with both $\Omega_{\rm core}$ and $\Omega_{\rm env}$ have these values connected by a dotted line. Figure constructed from data in Mosser et al. (2012), Deheuvels et al. (2012, 2014), Beck et al. (2014), Kurtz et al. (2014), and Saio et al. (2015).

The rotationally-split multiplet components of mixed dipole modes have similar probing power than in the case of hybrids with multiplets. Their ability to derive constraints on $\Omega(r)$ was first exploited by Beck et al. (2012), who studied several red giants after two years of *Kepler* monitoring. A large sample of red giants was meanwhile analysed by Mosser et al. (2012). These authors showed that dipole mixed modes mainly allow to deduce the core rotation, although inversion of the rotationally split multiplets in subgiants and giants in the early red giant branch also delivered limits on the envelope rotation (Deheuvels et al. 2012, 2014; Beck et al. 2014). We have $\Omega_{\rm core}$-values available for hundreds of subgiants and red giants determined by Mosser et al. (2012). All stars of masses between 0.7 and 3 $M_\odot$ whose interior rotation frequency was measured seismically have been assembled in Fig. 4. These seismic data shed new light on the theory of stellar evolution, because they deviate from it by two orders of magnitude in the red giant stage (e.g., Eggenberger, Montalbán & Miglio 2012; Cantiello et al. 2014). This implies that at least one physical ingredient that couples the stellar core to the envelope and that is active prior to the core helium-burning stage is missing, irrespective if the star was born with a convective or radiative core. Earlier on, seismic evidence for the loss of angular momentum prior to the white-dwarf stage had already been found (Charpinet et al. 2009). The red giant asteroseismology now shows that angular momentum transport must occur prior to the helium core burning stage for low to intermediate mass stars. Unfortunately, we do not yet have any insight into $\Omega_{\rm core}$ at stellar birth, as indicated by the "?" in Fig. 4.

To place the seismic results in Fig. 4 into a more global evolutionary picture and stress that the sample in Fig. 4 concerns only slow rotators and low- to intermediate-mass stars, we show in Fig. 5 several samples of unevolved galactic OBA-type stars with measured $\Omega_{\rm env}$, either from spectro-polarimetry or from asteroseismology. In case both a spectro-polarimetric and a seismic measurement of $\Omega_{\rm env}$ are available, they are fully compatible with each other. Figure 5 contains all the intermediate and high-mass stars that also have a seismically determined $\Omega_{\rm core}$. The red giants that did not undergo a helium flash at the onset of core helium burning but started it quietly after crossing the Hertzsprung gap are repeated from Fig. 4; all of these also had a convective core during the core hydrogen burning just as the other shown stars. Evolutionary tracks for a 5 $M_\odot$ star rotating at a few fractions of the critical velocity at birth and based on angular momentum transport in a diffusive approximation while including rotationally-induced mixing are shown to guide the eye; these models were taken from Brott et al. (2011a). Despite the fact that the sample shown in Fig. 5 is limited to stars for which $\Omega_{\rm env}$ could be measured, which excludes the fastest rotators as well as stars of much lower metallicity than the Sun, it covers a much broader range in surface rotation frequency than those in Fig. 4. There is fairly good agreement between the measured $\Omega_{\rm env}$ and the model tracks for various masses and rotation

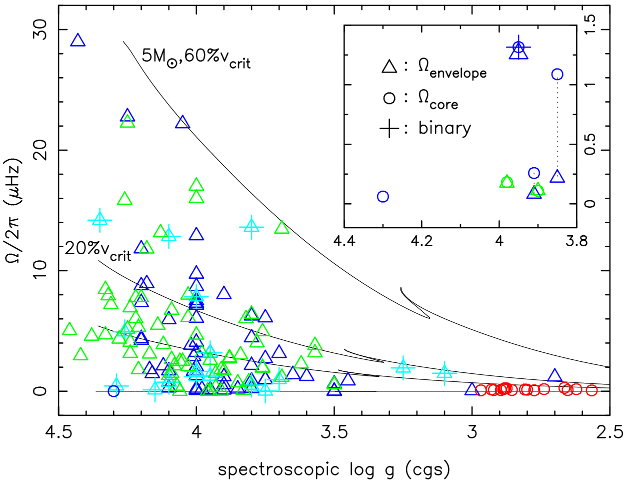

Fig. 5 The core and/or envelope rotation frequency of stars with birth mass above 2.25 and up to some 50 $M_\odot$ for which this measurement is available, as a function of their spectroscopically determined gravity as a proxy for their evolutionary stage. The error on the frequencies is much smaller than the symbol sizes. A typical uncertainty for the spectroscopic $\log g$ amounts to 0.15. Data for OB stars (dark blue: single; light blue with cross: binary) taken from Aerts et al. (2014), for Bp/Ap stars from Hubrig et al. (2007), for Vega from Petit et al. (2010), and for the two hybrid pulsators from Kurtz et al. (2014) and Saio et al. (2015) (green). The core helium-burning red giants with a mass above 2.25 $M_\odot$ from Mosser et al. (2012) are indicated in red. The inset shows the massive stars with seismically determined values of $\Omega_{\rm core}$. Four evolutionary tracks for a 5 $M_\odot$ star rotating at the indicated fractions of the critical value (the two lower ones valid for 10 % critical and without rotation) were taken from Brott et al. (2011a).

values given in Brott et al. (2011a). A general trend is that most of the massive stars for which $\Omega_{\rm env}$ could be measured, support the picture that their surface gets slowed down efficiently by the time they have exhausted their central hydrogen, somewhat independently of the surface rotation with which they were born. On the other hand, the processes that transport angular momentum in stars may also induce mixing and the occurrence of slowly rotating, nitrogen-enriched stars seems to point to shortcomings in the models in this respect (Brott et al. 2011b; Aerts et al. 2014).

Just as for red giants, stronger core-to-envelope coupling than foreseen in current state-of-the-art models, again with two orders of magnitude, is required to bring the models of massive stars in agreement with measured rotation rates of young neutron stars and white dwarfs (e.g., Langer 2012). It is thus tempting to assume that one efficient physical angular momentum transport mechanism is at work in all stars, yet is missing in current models. Given that the incidence of magnetic fields is, at best, limited to 10 % for O to F-type stars (e.g., Donati & Landstreet 2009; Petit et al. 2013), an excellent candidate mechanism could be internal gravity waves (e.g., Talon & Charbonnel 2005; Rogers et al. 2013). High-precision CoRoT and *Kepler* space photometry of the most massive stars supports this suggestion (e.g., Tkachenko et al. 2014 for a discussion).

4 Tidal asteroseismology

None of the above considered pulsations in close binaries. Asteroseismic studies of binaries, in particular eclipsing ones, are an asset for stellar physics because the binarity offers model-independent constraints on the masses and radii of the stars, while these parameters are outcomes of seismic modelling in the case of single stars. Isochrone fitting of double-lined eclipsing binaries offers an independent way to tune interior structure parameters, such as core overshooting (e.g., Torres 2013). For young massive stars this gives compatible estimates with seismic modelling of single pulsators (e.g., Fig. 2 in Aerts 2015).

Obviously, a combination of binary and asteroseismic modelling has the potential to put tighter constraints on the stellar structure evaluation if the pulsator is a member of a binary, even if tides are not involved. This benefit was already demonstrated for the α Cen system (e.g., Miglio & Montalbán 2005) and for the above mentioned sdB pulsator PG 1336–018 (Van Grootel et al. 2013) from ground-based asteroseismology and is being applied to space photometry now as well (e.g., Southworth et al. 2011; Frandsen et al. 2013, Maceroni et al. 2013, Beck et al. 2014; Gaulme et al. 2014; Boumier et al. 2014). A recent overview of asteroseismology in eclipsing binaries with focus on solar-like pulsations is available in Huber (2014). It is noteworthy that binary light curve modelling tools developed in the mmag-precision photometry era are not able to deal with the current μ mag photometric precision. The space photometry revolution thus triggered the development of new software tools to handle the binary modelling at the level of the precision of the current data (Prsa et al. 2013; Degroote et al. 2013; Bloemen et al. 2013).

The subject of *tidal asteroseismology*, i.e., stellar modelling based on tidally excited or tidally affected pulsations, only turned into a practical science since the availability of the uninterrupted CoRoT and *Kepler* lightcurves of either eclipsing binaries with tidally excited g-modes (e.g., Maceroni et al. 2009; Welsh et al. 2011; Hambleton et al. 2013; Debosscher et al. 2013; Borkovits et al. 2014) or pulsating stars that were initially thought to be single stars but turned out to be a member of a spectroscopic binary from follow-up data (e.g., Pápics et al. 2013). In all of those studies, clear evidence was found that some or all of the g-modes are tidally triggered, after iterative light curve modelling schemes in the cases where the binary and pulsational variability are of the same order of magnitude. Modelling of tidally excited pulsation modes triggered by dynamic tides active in eccentric binaries offers the opportunity to put stringent constraints on the interior structure models of the binary components by exploiting the tide-generating potential. This was so far only worked out in detail for the most eccentric pulsating *Kepler* F-type binary KOI-54 by Fuller & Lai (2012).

Tidal asteroseismology has immense potential. The photometric observational data have been assembled and are being analysed in terms of pulsation frequencies and mode identification while spectroscopic orbital monitoring is ongoing for tens of systems, by various research teams. Full exploitation of the tide-generating potential will require new work-intense theoretical developments because the modelling needs to be tuned to each of the binary pulsators individually.

5 Future prospects

The past five years have delivered tremendous progress in asteroseismology. Thanks to the long-term uninterrupted high-precision space photometry, we have moved from the detection of a few tens of pulsation modes in a handful of solar-like stars to hundreds of modes in thousands of single and binary stars of very diverse evolutionary stage. Despite great achievements, some of which discussed above, the best of asteroseismology for stellar physics is yet to come. Indeed, few of the four-year *Kepler* light curves have been exploited in full detail in terms of forward modelling based on individual frequency matching. While the used scaling relations for solar-like pulsations are of great value, they can never capture the fine details of the interior structure for each and every individual star, as forward seismic modelling can. The pulsation frequencies are now measured with such high precision that the forward modelling process necessarily has to follow an iterative scheme, where improvements on the input physics of the stellar models are mandatory to bring the data in agreement with the theory. Such scheme can only be automated to a limited extend and requires detailed tweaking, as it was done so beautifully in Helioseismology (e.g., Christensen-Dalsgaard 2002). Similar efforts for stars will allow deep understanding of currently poorly described physical processes inside stars, several of which inaccessible for the Sun because it only reveals p-modes and is a relatively unevolved slow rotator.

A badly known piece of information in stellar evolution theory that we have already briefly touched upon concerns the star formation process, the end of which is taken as the initial condition for stellar evolution computations. Star formation is a notoriously difficult subject, both from a theoretical and an observational point of view. The final phases of the star formation process not only determine the starting point for the life of the star, but are also highly relevant for exoplanetary system formation and evolution. While the contraction of protostars towards hydrogen ignition is known in a global sense (e.g., Palla & Stahler 1992), details about the matter accretion and energy dissipation are yet to be investigated and calibrated. Also here, asteroseismology has a role to play, given that pre main-sequence (PMS) stars of intermediate mass are expected to undergo p-mode pulsations (Marconi & Palla 1998). PMS asteroseismology has hardly been explored so far, because such targets did not occur in the *Kepler* nominal field-of-view and they were poorly sampled with CoRoT. Nevertheless, Zwintz et al. (2014) recently came up with the first effort to age PMS stars seismically, by considering the highest detected p-mode frequency in MOST (Matthews 2007) and CoRoT data of young open cluster stars as a good proxy for the *acoustic cutoff frequency*. This theoretical quantity is tightly connected with the way the p-modes are damped by outwardly propagating running waves in the upper atmosphere of the star (e.g., Hansen et al. 1985) and may thus tune the physical conditions in the atmosphere of PMS stars. As a side note, we remark that a similar quantity applies at the other extreme of the longest detectable g-mode period (Townsend 2000a,b). As notably visible in Fig. 2, rotation strongly affects this g-mode period cutoff. This has so far remained unexploited in practice for g-mode pulsators. To investigate if the PMS phase is of importance for the life of stars and their planets, such stars have been proposed for observations with the K2 mission (Howell et al. 2014).

Although K2 can only deliver light curves with a duration of some 75 d, with somewhat less precision than the nominal *Kepler* mission, its great asset is that it can point to various observing fields in the ecliptic, thus delivering seismic data for a much larger variety of stars in terms of mass, metallicity, and evolutionary stage. This holds the potential to cover the entire Hertzsprung-Russell diagram with asteroseismology based on pulsational phenomena with mode periodicities ranging from a few minutes to several weeks. The K2 data cannot deliver $\Omega(r)$ for low-mass stars but offers the opportunity to determine this quantity for massive stars, from rotational splitting of their p-modes.

With launch foreseen in 2017, the TESS mission (Ricker et al. 2015) will provide high-precision photometry at 2-min cadence for stars with I magnitude in the range 4 to 13. TESS will operate during two years and has all-sky coverage. Its light curves will be limited to typically a month duration, except for the ecliptic poles, which will be monitored during a full year. Hence we look forward to even more diversity in asteroseismic applications, including the Magellanic Clouds!

Even farther in the future, the PLATO mission up for launch in 2024 (Rauer et al. 2014), will deliver long-term high-precision photometry at 50 s cadence in white light, but with the capability of two-colour photometry for the brightest targets. For a modest number of targets, monitoring with fast cadence of seconds is foreseen. PLATO data will cover two yet to be chosen huge fields-of-view (2250 square degrees). Unlike any of the previous space missions, PLATO is specifically designed to have simultaneous asteroseismology and exoplanet detection capabilities. PLATO will be a 6-year mission, with two long-duration pointings of two and three years each, and a final year with step-and-stare option to (re-)visit various fields all over the sky during several weeks to months. Indeed, the best is yet to come.

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