In Search of the
Low Surface Brightness Universe

Jonathan I. Davies

Department of Physics and Astronomy,
Cardiff University, Wales, UK

Abstract

In this paper I discuss how the study of Low Surface Brightness (LSB) galaxies has developed over the last 10 years. In particular I concentrate on the influence the discovery of these galaxies has had on the faint-end slope of the luminosity fuction and our understanding of the distribution of surface brightness. Although it is still unclear how LSB galaxies affect our understanding of field galaxies it is clear that LSB galaxies dominate both the total luminosity and absorption cross-section in clusters.

1 Introduction

> *'We are like prisoners in a lighted cell trying to discern our whereabouts by peering out through a small casement into the darkness outside. We can see the street lamps easily enough, and the lighted windows, but can we see or correctly infer the houses and the trees.'* *Disney 1980*

The above statement is a poignant reminder to all observational astronomers that what we observe may be more a consquence of our position in the Universe than of its true physical form. It is an anthropic statement about the inevitable conclusions one must draw from observing the Universe from our present position within the disc of a luminous spiral galaxy close to a rather average main sequence star – the extra-galactic Universe is not easy to see. Most of the projected surface area of a 'typical' galaxy is fainter than the night sky as viewed from the surface of the Earth, and this is true over the entire optical and near infrared parts of the spectrum (The darkest night sky has a surface brightnesses of about 23 $B\mu$ (Blue magnitudes per sq arc sec) while the average surface brightness of a 'typical' galaxy (within its half-light radius) is about 23.5 $B\mu$).

Arp (1965) was the first to show that there was possibly a severe bias against galaxies with both high and low surface brightness. In a plot of magnitude against size he showed that a wide range of very different galactic objects followed a line of almost constant surface brightness. In 1970 Freeman noted that a sample of some of the largest angular size spiral galaxies indeed had approximately the same surface brightness (central surface brightness $\mu_o^B \approx 21.7\ B\mu$). Freeman's interest was really in the dynamics of these spiral galaxies and not in their surface brightness and he did not, at that time, pursue the matter. In 1964 Fish had also been interested in galaxy dynamics, but in this case the dynamics of elliptical galaxies. He noted that the binding energy of an elliptical galaxy varied as the 3/2 power of the mass, a result that at first-sight does not seem to relate to surface brightness. Disney (1976) showed that in fact Fish's relationship implied that elliptical galaxies also have approximately the same surface brightness ($\mu_o^B \approx 14.8\ B\mu$). What intrigued Disney was that he could show that for a given limiting (detection) isophote (μ_L^B) and at a fixed absolute magnitude a spiral galaxy would subtend a maximum angular size when $\mu_L^B - \mu_o^B \approx 2.2$ and an elliptical galaxy when $\mu_L^B - \mu_o^B \approx 9.1$ (the different values arise because of the different surface brightness profiles of the two types of galaxy). These values implied μ_o^B values of 21.8 and 14.9 for spiral and elliptical galaxies respectively if $\mu_L^B = 24$, which it was for the typical photographic data of the day.

This result indicated to Disney that Freeman's and Fish's result (constant surface brightness) was purely a result of observational selection – only those galaxies of a preferred surface brightness, that subtend a maximum angular size are selected for. In essence High Surface Brightness (HSB) galaxies appear small because they are compact while, Low Surface Brightness (LSB) galaxies appear small because μ_0^B is close to μ_L^B. This started a number of controversies that have to some extent lasted until today. These are:

1. If galaxies all have the same surface brightness (it is not a selection effect) then why?
    (a) Is it something to do with the way surface brightness is defined?
    (b) Is it due to dust opacity?
    (c) Is this an important clue to further our understanding of the nature of galaxies (formation/evolution)?
2. If Freeman and Fish's results are due to observational selection
    (a) Do LSB galaxies exist?
    (b) Do compact HSB galaxies exist?
3. If HSB and/or LSB galaxies exist what is the number/luminosity/mass density of the galaxies that have not yet been catalogued?

Since 1976 great progress has been made in deciding what is the true nature of the galaxy population of the Universe. I will describe some of this work in the following sections concentrating on the LSB galaxy issue.

2 Is Freeman's result universal ?

In this section I will briefly describe why I believe items 1a and b above can be discounted, 1c is a bit more complicated. Both Kormendy (1977) and Phillipps and Disney (1983) have discussed in detail how the fitting of a combined $r^{1/4}$ (bulge) and exponential (disc) surface brightness profile may lead to Freeman's result irrespective of the 'true' disc surface brightness (see also Davies 1990). I believe that this is really a 'red herring' as there are clearly many spiral galaxies that have almost perfect exponential profiles yet still have central surface brightness close to Freeman's value. In addition, as discussed below, many LSB spiral galaxies (with bulges) have now been found.

Is it something to do with the way galaxies have formed and evolved ? This is more intriquing. In its simplest interpretation the Tully-Fisher relation can be written as

$$v \propto L^{1/4}\Sigma^{1/4}(M/L)^{1/2} \tag{1}$$

which for constant surface brightness (Freeman) and (M/L) becomes the observed

$$v \propto L^{1/4} \tag{2}$$

Recently the Tully-Fisher relation for LSB galaxies has been shown to lie on the same line as that for the brighter galaxies (McGaugh and de Blok, 1998a). This can only be true if (M/L) and Σ conspire with each other so that their product remains constant. What this means in terms of galaxy formation/evolution is not at all clear. It could just mean that you find high values of (M/L) for galaxies that have not yet converted much material into stars, hence their LSB. This requires some fine tuning, particularly with the dark matter, though there are many indications that LSB spiral galaxies have inhibited star formation (van der Hulst, 1993, McGaugh and Bothun, 1994, de Blok et al. 1995). McGaugh and de Blok (1998b) have used this Tully-Fisher result to argue for non-Newtonian gravity (MOND).

We have spent some time investigating the opacity issue (see Davies and Burnstien 1995 and references therein). An explanation of the constancy of surface brightness could be that spiral discs are optically thick over a good fraction of their surface area. This would result in the surface brightness being determined by the typical mean free path of a photon rather than the total column of stars through the disc (Disney et al. 1989). This issue was fueled further with Valentijn's (1990) claim that galaxies were optically thick at optical wavelengths out to large radii (see also Burnstein et al. 1991). We extensively analysed Valentijn's and Burnstein's methods and interpretation (they inferred the opacity from the statistical variation of surface brightness with viewing angle) and concluded that it was impossible to disentangle the opacity issue from the observational selection effects affecting every sample of galaxies (Davies et al. 1995, Jones et al. 1996). We also concluded that the only way to assess the opacity of a spiral galaxy was to measure the fraction of the stellar light reprocessed through dust. This meant comparing the optical and far-infrared outputs of galaxies. This work goes on, but our conclusion seems to be that the optical surface brightness of a 'typical' galaxy is not on the whole determined by optical depth effects (Alton et al. 1998, Bianchi 1999). In the main it appears that a 'typical' galaxy is reprocessing about 50 % of its optical radiation through dust, but the dust is very patchy.

Having discounted other explanations of Freeman's result and considering the recent discovery of many LSB galaxies (see below) I now believe that Freeman's result was a selection effect. He did no more than select the most prominant galaxies for a dynamical study. The problem now is to assess how important LSB galaxies are.

3 The luminosity and surface brightness functions

One of the fundamental observable quantities of galaxies is the Luminosity Function (LF). From this we can integrate to obtain the luminosity density of the Universe due to stars in galaxies and multiply by a suitable value of (M/L) to obtain the mass density. In essence Disney was saying that this function was incomplete because in all likelihood there were many galaxies that had not yet been detected. This incompleteness can be understood and quantified by using a Bi-variate Brightness Function (BBF) instead. For example the number of galaxies per Mpc^{-3} per unit luminosity and surface brightness (Davies 1990, de Jong 1998). Knowing the BBF rather than just the LF enables us to answer questions like:

1. Where does most of the luminosity of the Universe come from? From faint galaxies, from bright galaxies, from LSB galaxies, from High Surface Brightness (HSB) galaxies?
2. Which kind of galaxies, along any arbitrary line of sight, are most likely to cause absorption features in, say, a QSO spectrum?

The BBF, $n(L, \Sigma)$ [1] [here I choose to use luminosity (L) and surface brightness (Σ)] is the number of galaxies per Mpc^{-3} per unit Σ and L. Assume that the population in L and Σ can be described by the distribution function

$$n(L, \Sigma) = N_o L^{\alpha} \Sigma^{\beta}$$

and that L and Σ are independent. With $\beta = 0$ (no SB dependence) this is a good approximation to the observed LF, i.e. the Schechter function with no bright end cut-off.

We can now ask the question: which galaxies contribute most light? Let T_L equal the total light from all galaxies so that

$$T_L = N_o \int_{L_{min}}^{L_{max}} \int_{\Sigma_{min}}^{\Sigma_{max}} L^{\alpha+1} \Sigma^{\beta} \quad \mathrm{d}L \mathrm{d}\Sigma$$

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[1] For the rest of this paper I will use μ to represent SB in magnitudes per sq arc sec and Σ to represent solar luminosities pc^{-2}

$$= \frac{N_o}{(\alpha+2)(\beta+1)}(L_{max}^{\alpha+2} - L_{min}^{\alpha+2})(\Sigma_{max}^{\beta+1} - \Sigma_{min}^{\beta+1})$$

If we take $L_{max} \gg L_{min}$ and $\Sigma_{max} \gg \Sigma_{min}$ then

Case $\alpha < -2$ – Light is dominated by low luminosity galaxies.
Case $\alpha > -2$ – Light is dominated by high luminosity galaxies.
Case $\beta < -1$ – Light is dominated by LSB galaxies.
Case $\beta > -1$ – Light is dominated by HSB galaxies.

What about the cross sectional area (A) these galaxies subtend on the sky? As $\Sigma = L/A$ the total area subtended is

$$T_A = N_o \int_{L_{min}}^{L_{max}} \int_{\Sigma_{min}}^{\Sigma_{max}} L^{\alpha+1}\Sigma^{\beta-1} \quad \mathrm{d}L\mathrm{d}\Sigma$$

$$= \frac{N_o}{(\alpha+2)(\beta)}(L_{max}^{\alpha+2} - L_{min}^{\alpha+2})(\Sigma_{max}^{\beta} - \Sigma_{min}^{\beta})$$

and as above

Case $\alpha < -2$ – Area is dominated by low luminosity galaxies.
Case $\alpha > -2$ – Area is dominated by high luminosity galaxies.
Case $\beta < 0$ – Area is dominated by LSB galaxies.
Case $\beta > 0$ – Area is dominated by HSB galaxies.

Before the current interest in LSB galaxies the 'canonical' values of α and β were -1.0 (a flat LF) and 0.0 (approximately constant surface brightness) respectively. Both the luminosity density and the absorption cross-section were dominated by HSB luminous galaxies. In the following sections I will describe how these values have changed over the last few years and why LSB galaxies are becoming an ever increasingly important constituent of the known Universe. I will start by discussing cluster and field galaxies separately.

4 The total numbers and luminosity density due to LSB galaxies in clusters, groups and the field

Historically there has been two types of survey for LSB galaxies and I believe that this has led to some confusion about the numbers of LSB galaxies and their nature. There have been surveys that have concentrated on covering large areas of sky looking for the LSB counterpart to the spiral galaxy population (see for example Impey et al. 1996) and there have been those that have concentrated on the LSB populations of nearby clusters (Impey et al. 1988, Irwin et al. 1990, Phillipps et al. 1998, Kambas et al. 1999). The cluster surveys invaribly find comparatively large numbers of LSB galaxies, but almost all of these are dwarf elliptical (dE) galaxies. Thus the two types of surveys have found, and, in some sense, been directed at finding different types of galaxies.

4.1 The Local Group

The present inventory of the Local Group (LG) consists of some 40 galaxies (Mateo 1998). The process of finding LG LSB galaxies is in many ways quite different to finding galaxies in other groups and clusters. Invariably LSB companions to the Milky Way have been identified by enhancements in the stellar number counts because they are resolvable into stars (see Irwin and Hatzidimitriou 1995 and references therein). The search area has to be large because the depth (hence volume) is small. To enhance the chances of success most searches have been concentrated on areas around the larger galaxies (Armandroff et al. 1999). The three major spiral galaxies appear to be 'typical' galaxies, most of the others are companions to the major spirals (a selection effect ?) and are typically LSB dwarf galaxies (the obvious exception being M32). The LF of the LG is flat ($\alpha \approx -1$, van der Bergh 1992, Mateo 1998). Using the data of Mateo (1998) (table 3) I can crudely estimate (what are the selection effects ?) a value of $\beta \approx -1.2$.

4.2 Local clusters

The first systematic listing of LSB dwarf galaxies in the nearby Virgo cluster was carried out by Reaves (1983). This was soon superceded by the comprehensive survey of Binggeli and collaborators (Binggeli et al. 1984, 1985, 1987, Sandage et al. 1985a/b). This work made use of the newly available large plate scale du Pont photographic plates. The survey listed some 1277 galaxies (stated completeness to $m_B = 18.0$ with $\mu_L^B \approx 25.5$) as cluster members, the vast majority being dwarf galaxies($M_B > -16$) of 'lowish' surface brightness ($\mu_o \approx 23 - 24\ B\mu$). The data were used to address a number of important issues. The LF of all morphological types was shown to be steeper ($\alpha \approx -1.4$) than that normally measured for field galaxies ($\alpha \approx -1.0$) (Loveday et al. 1992, Lin et al. 1996). In addition because of the quality of the data the LF for different morphological types of galaxies could be determined. This clearly showed the Gaussian nature of the LF of spiral and elliptical galaxies (Hubble 1936) and the power law LF of the dE galaxies (Zwicky 1957). In addition they showed that the dE galaxies followed a surface brightness luminosity relation of the form $\mu_o^B = 0.7M_B + 34.0$ (taken from Ferguson and Binggeli 1994). Folding this into the LF leads to a value of −1.6 for β (but remember the calculation in section 4 assumed L and Σ were independent).

The observed surface brightness magnitude relation was criticised by Phillipps and Davies (1988) because the influence of the selection criteria had not been properly considered. Their concern was confirmed when Impey et al. (1988) extended the Virgo observations to lower surface brightnesses by using 'Malinized' (contrast enhanced) photographic plates. Essentially they found lower surface brightness galaxies over a range of magnitudes. This work also indicated that significant numbers of galaxies that the Binggeli survey should have detected given the magnitude limit, were in fact missing because their surface brightness was too low. Including these galaxies steepened the

LF to $\alpha \approx -1.7$. The surface brightness luminosity relation has not gone entirely away though, even after due consideration of the selection effects. I now believe that there is a loose relationship between surface brightness and luminosity, but it is nowhere near as tight as first proposed by Binggeli et al. In a recent survey for dwarf galaxies ($M_B > -11.5$) in the Fornax cluster Kambas et al. (2000) find that galaxies of fainter apparent magnitude do tend to have lower surface brightness.

More recent observations of Virgo (Phillipps et al. 1999) have been used to derive, statistically, the LF of the Virgo cluster (they use fields outside the cluster to remove background galaxies from the Virgo sample). These observations now extend to $M_R = -10$ and $\mu_o^R = 25.0$. Phillipps et al. derived a value of $\alpha \approx -2.2$, but make no comment on β. I have used the Phillipps et al. and the Binggeli et al. data to make a crude estimate of β for the Virgo cluster. There are about 800 dE with $\mu_o^B \approx 23$ listed by Binggeli et al., while the Phillipps et al. data implies about 30000 Virgo galaxies with $\mu_o^B \approx 25$. This leads to $\beta \approx -3.0$. Although a very crude estimate it is clear that the Phillipps et al. result implies both a large value for α and β. The above is summarised in the table below.

Virgo surveys	**Limiting Surface brightness (B)**	α	β
Binggeli et al. (1984)	25.5	–1.4	–1.6
Impey et al. (1988)	26.5	–1.7	?
Phillipps et al. (1998)	27.7	–2.2	–3.0

The other well studied nearby cluster is the elliptical rich Fornax cluster. The first comprehensive study of Fornax was carried out by the Cardiff group (Cawson et al. 1987, Phillipps et al. 1987, Davies et al. 1988, Kibblewhite et al. 1989, Irwin et al. 1989, Disney et al. 1990). Relevant results from this work include a catalogue of some 300 LSB galaxies that were thought to be cluster members and a measurement of $\alpha \approx -1.5$ and $\beta \approx -1$. During this time Ferguson (1989) also produced a catalogue of Fornax cluster members using similar plate material to the Bingelli et al. Virgo survey. He measured a LF faint-end slope of $\alpha \approx -1.3$.

More recently we have used CCD observations of Fornax to extend the photographic work to some two magnitudes fainter in surface brightness (Kambas et al. 1999). We were able to define the cluster population because the data extend to large distances from the cluster centre and so we can measure the decreasing surface density of galaxies. Our result leads to $\alpha \approx -1.8$ and $\beta \approx -3.0$ (this has been calculated in the same crude way as I did for the Phillipps et al. Virgo data). This result is remarkably consistent with the Phillipps et al. data for Virgo. The Fornax observations are summarised in the table below.

Fornax surveys	**Limiting Surface brightness (B)**	α	β
Davies et al. (1988)	25.5	–1.5	–1.0
Ferguson et al. (1989)	25.5	–1.3	?
Irwin et al. (1989)	26.5	–1.5	–1.0
Kambas et al. (1999)	27.5	–1.8	–3.0

For other more distant clusters it is more difficult to trace the galaxy population to such low luminosities and surface brightness levels. Trentham (1997a/b) used the statistical subtraction of background fields method to determine the LF of 8 clusters (down to $M_R \approx -10$). For four elliptical rich clusters he found $-1.5 < \alpha < -1.2$ and for four spiral rich clusters a steeper slope $-1.8 < \alpha < -1.6$. Muriel et al. (1998) derived the LF of a number of clusters and groups. These observations extend down to $M_B \approx -16$ and to surface brightness of $\mu_o^B \approx 23$ (assuming typical photographic surface brightness levels). This is not really faint enough to pick up the LSB population that gives rise to the steep faint-end slope in Virgo and Fornax, but they do find a clear difference between groups and clusters. The groups tend to have much flatter LFs ($\alpha \approx -1$), very similar to the LG, while the clusters have a steeper LF ($\alpha \approx -1.4$). Neither Trentham nor Muriel provide information that can be used to derive β.

4.3 Field galaxies

To carry out a similar analysis, to that described above, for field galaxies is much more difficult. The volume density of galaxies is obviously lower in the field (also true for LSB galaxies ?) so larger areas have to be surveyed to obtain statistically significant samples. Even then there is ambiguity over whether the galaxies are nearby or cosmologically dimmed distant galaxies. In most cases a distance is required to decide on the actual surface brightness and to assess the contribution to the LF. Redshifts are difficult to obtain optically because of the low surface brightness, though 21 cm redshifts are possible as long as the galaxies are relatively gas rich (the numerically dominant dE galaxies in clusters are gas poor). For these reasons the first field surveys concentrated on LSB gas rich galaxies (Impey et al. 1996). The Impey et al. survey detected some 693 LSB (mainly late type) galaxies over 786 sq deg. The detection isophote was $\approx 26\ B\mu$ and they found galaxies with surface brightnesses almost to this limit. In a subsequent paper Sprayberry et al. (1997) used this data to derive the LF of LSB galaxies. They derive a value for α of –1.4. This compares with a 'standard' picture of the LF of the field which gives $\alpha \approx -1$ (Loveday et al. 1992, Lin et al. 1996). Given the normalisation of the two LFs there is about 1/3 of the luminosity of the bright galaxies in the LSB population.

Two recent determinations of the field galaxy LF have extended to faint absolute luminosities, but not necessarily faint surface brightness. Huchra (1999) has discuss the LF derived from the CfA2 redshift survey. Reading from his figure 1. I derive a value of $\alpha \approx -1.5$. The remarkable value found by Loveday (1997) is $\alpha \approx -2.2$. To try and reach very low surface brightnesses ($\mu_o^B \approx 26$) Davies et al. (1994) used a large format CCD to place limits on the numbers of field LSB L* galaxies. They concluded that they are much rarer than 'normal' L* galaxies.

Almost all of the above work has concentrated on LSB gas rich spirals. Yet, in clusters the most prolific LSB galaxy is the dE. Could these contribute significantly to the field galaxy LF ? We have recently carried out two surveys that suggest that they do not. Morrshidi-Esslinger et al. 1998a/b used over 60 photographic plates to search some 2400 sq deg of sky for dE galaxies typical of those found in nearby clusters. Objects were detected down to $m_B \approx 20$ and $\mu_o^B \approx 23.5$. The result was that the dE galaxies are essentially confined to the clusters and groups. Although their clustering scale is more extended than the brighter galaxies they still follow the same large scale structure. This result has been confirmed by the recent CCD survey of Kambas et al. 2000. Their result for the Fornax cluster, as described above, also showed that the dE galaxies were predominantly cluster galaxies and are not prolific in the field. The field subtraction methods (Phillipps et al. 1998, Trentham 1997a/b) also confirm the lack of low luminosity galaxies in the field – they find a large excess of LSB galaxies in the cluster fields.

Both Davies (1990) and McGaugh (1995) have considered the surface brightness distribution of field LSB galaxies. After correcting for the volumes sampled at each surface brightness both concluded that the surface brightness distribution of the field galaxies was flat ($\beta \approx -1.0$). More recently O'Neil and Bothun (2000) have confirmed this result. The above discussion is summarised in the following table.

Field surveys	**Limiting Surface brightness (B)**	α	β
Loveday et al. (1992)	24.5	–1.0	
Impey et al. (1996)	26.5	–1.4	
Loveday (1997)	24.5 ?	–2.2	
Huchra et al. (1999)	24.5 ?	–1.5	
Davies et al. (1990)	25.5	–	–1.0
O'Neil and Bothun (2000)	25.0	–	–1.0

5 Summary

What is clear to me from the above is that both α and β have gradually been increasing as more and more LSB galaxies have been discovered. I am now convinced that both the total luminosity density (Kambas et al. 1999) and the absorption cross-section (Impey et al. 1999) of cluster galaxies is dominated by the low luminosity LSB population. This is particularly interesting because recent work indicates that both low luminosity and LSB galaxies invariably have a large fraction of their mass as dark matter (McGaugh and de Blok, 1998b, Carignan and Freeman 1988). Thus they may well be the site of the cluster dark matter. The large values of α indicate a divergent LF so it must turn down or terminate beyond about $M_B = -10$.

Given that only about 10 % of galaxies reside in clusters (Fukugita et al. 1998) the values of α and β for field galaxies are much more interesting. At present I find a lot of the information about the field galaxy population confusing. For example if Loveday's (1997) LF is correct then the survey of Morshidi-Esslinger et al. (1999a/b) should have found many more dwarf galaxies in the local field and the cluster field subtraction methods should not work. We find a large decrease in LSB galaxy numbers as we go from the cluster to the field (Kambas et al. 2000), where are all these faint field galaxies? Ferguson (1999) has used the HDF to place limits on the LSB galaxy population and excludes large numbers of 'normal' luminosity LSB galaxies, though he cannot exclude large numbers of low luminosity galaxies.

The LSB field population awaits a definitive answer, but I suspect that there are nowhere near the same relative numbers of low luminosity LSB galaxies in the field as there are in the clusters, but there is still the possibility that a population of large very LSB galaxies exists. For example, the largest spiral galaxy known is the giant LSB spiral galaxy Malin 1 (Bothun et al. 1997). Given its very LSB, $\mu_o^B \approx 26.5$, and large size (scale size of ≈ 50 *kpc*) it is not surprising that it was really a chance detection (it has a higher surface brightness bulge). Present optical surveys have really found it very difficult to reach the surface brightness levels necessary to detect galaxies like Malin 1. Some of the new large format CCD surveys along with follow up multi-object spectroscopy may reveal the field LSB galaxy population to us over the next few years. Objects like Malin 1 are also gas rich so maybe the currently on going 21 cm all sky surveys at Parkes and Jodrell Bank will place firm constraints on the numbers of gas rich LSB galaxies that remain hidden from optical surveys.

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