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Cosmic wave-particle interactions: Astrophysical magnetic turbulence and high-energy particles

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Understanding the origin and the properties of energetic charged particles has been an important problems of both plasma astrophysics and astroparticle physics. Here, an introduction is given about the complicated interaction processes between cosmic rays and turbulent electromagnetic fields. First, the formation of turbulent electromagnetic fields due to plasma instabilities is discussed. Second, the microphysics of particle scattering due to such turbulent fields is explained and a comparison with measurements in the Solar wind is shown.

1 Introduction

This article will attempt to provide a brief summary of recent innovations for describing the fascinating interplay of high-energy particles and turbulent magnetic fields. Both in theoretical and observational (plasma) astrophysics, such is important for a number of reasons (see Schlickeiser 2002, for an introduction). A few examples are found in the questions (i) of how cosmic rays are accelerated; (ii) of the role of galactic winds and supernova-induced turbulence; (iii) of solar flares and coronal mass ejections – which Earth and especially electronic devices experience as the so-called “space weather” (Scherer et al. 2005).

From a more theoretical point of view, these problems have to be tackled in terms of wave-particle interaction processes as known from plasma physics. For both historical and practical reasons, these have usually been divided into two different approaches, which correspond to two fundamental questions: (i) When is a plasma unstable so that turbulent fields are generated? (ii) Once the plasma is saturated: how do the particles move? Aside from energetic particles in the heliosphere, there are widespread applications ranging from laboratory and fusion plasmas to the interstellar medium and magnetic field generation in the early universe – in fact, wherever charged particles move in a magnetized environment, the principles presented here apply (even though extreme parameter regimes may require specialized methods). In the heliosphere, however, we are especially lucky as we can measure the behavior of both particles and electromagnetic fields and so put our theories to the test.

The history of high-energy astrophysics began in in the first decades of the 20th century, when, with increasing altitude, *more* ionizing radiation was found – instead of less, as people expected the radiation to originate in the Earth’s crust. The picture of Viktor Hess in his balloon, from which he took his measurements, is famous among cosmic-ray physicists. At the time, however, various people were interested in this (see, e. g., Carlson 2012) and so even the term “cosmic rays” was coined at least twice.

Today we know that cosmic rays are in fact a stream of particles that arrive almost isotropically on Earth. They consist predominantly of protons and alpha particles (around 90 % and 9 %, respectively) with the rest being electrons, heavier nuclei, and very few anti-particles (if any). One of the most striking features that characterizes cosmic rays is the broken, but otherwise extremely smooth, power law that the flux exhibits as a function of the particle energy. It extends over a huge energy range, which is even more remarkable as the sources are very different, depending on the respective energy bands:

1. up to 10^{10} eV: the Sun during energetic solar particle events due to magnetic reconnection (Holman 2012);
2. up to 10^{15} eV: galactic sources, mostly shock waves from supernova remnants (Arons 2003);
3. up to 10^{20} eV: extragalactic sources such as active galactic nuclei (Biermann & de Souza 2012) and perhaps gamma-ray bursts.

For the highest energies, the count rates drop dramatically, but the cut-off at $\sim 10^{20}$ eV that had been expected from theoretical arguments (GZK cutoff) now seems to have been confirmed (e. g., Letessier-Selvon & Stanev 2011).

When one attempts to study the mechanisms that produce these cosmic particles, one first needs to understand cosmic magnetic fields as these are omnipresent in the universe: from the Earth, where the strong magnetic field induced by a dynamo that is kept running by the moon, to the solar magnetic field that extends throughout the heliosphere

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in the form of famous Parker spiral (Parker 1958), to Galactic (Beck et al. 1996) and extragalactic magnetic fields. For the case of the heliosphere and the Milky Way, typical field strengths are of the order of a few micro-Gauss to nano-Tesla. In most cases, the magnetic fields can be divided in two categories, namely regular, large-scale fields; and turbulent, small-scale fields. The critical point is that often both components are of the same order of magnitude, thereby disallowing to neglect either one. Instead, in many cases the simplifying assumption is made that the large-scale component can be regarded as homogeneous so that one can write $\boldsymbol{B} = B_0 \hat{\boldsymbol{e}}_z + \delta \boldsymbol{B}$.

The main reason why magnetic fields – especially if they are turbulent – are so important is their capability to deflect charged particles. Accordingly, attempts to understand the origin of cosmic rays with high energies being in supernovae had to rely on indirect methods (Aharonian et al. 2004). Similarly, for predictions of the time and location when and where a coronal mass ejection reaches Earth, its motion has to be traced through the solar-wind induced turbulence in the heliospheric magnetic field. This article provides an overview over both the basic theoretical methods and recent developments together with some results.

2 Electromagnetic turbulence

Understanding the origin of turbulent electromagnetic fields from a microphysical point of view first requires a reservoir of free energy that can be converted into turbulence. For this, there are two main possibilities:

1. directed particle streams, either into a medium at rest or in the form of counterstreaming plasma components;
2. anisotropic and/or inhomogeneous temperatures as expressed through the mean kinetic energy of a particle ensemble.

Measurements of the electron velocity distribution function in the solar wind indeed show a large degree of anisotropy (Marsch 2006; Marsch et al. 1982). Accordingly, the onset of an instability in such an environment is to be expected.

In the case of tenuous plasmas such as the interplanetary and interstellar medium, the founding principle behind the generation of turbulence is the kinetic plasma theory. An important example is found in application to unmagnetized relativistic plasmas (see Schlickeiser 2010; Tautz & Lerche 2012, and references therein), where the generation of small-scale magnetic fields has been considered a possible mechanism for seed magnetic fields (Sakai et al. 2004; Schlickeiser 2005). A special class of such instabilities is those resulting in *aperiodic* modes, which can be characterized as purely growing without wave propagation.

From a technical point of view, in all these cases dispersion relations are derived from the linearized linear Vlasov equation, which, under the assumption of spatial homogeneity, requires the specification of an initial equilibrium velocity distribution function. A typical (non-relativistic) dispersion relation for wave vectors parallel to a given symmetry axis reads

$$\omega^2 = c^2 k_\parallel^2 + \sum_a \omega_{\mathrm{p},a}^2 \left[1 - \pi k_\parallel \int_{-\infty}^{\infty} \mathrm{d}v_\parallel \int_0^\infty \mathrm{d}v_\perp \right. \\ \left. \times \frac{v_\perp^3}{\omega - k_\parallel v_\parallel} \frac{\partial f(v_\parallel, v_\perp)}{\partial v_\parallel}\right], \tag{1}$$

where $\omega_{\mathrm{p},a} = \sqrt{4\pi n_a q_a^2 / m_a}$ is the plasma frequency for particle species a and where $f(v_\parallel, v_\perp)$ denotes the gyrotropic velocity distribution function. For the typical case of a Maxwellian plasma, Eq. (1) has to be solved in terms of the plasma dispersion function (Tautz & Schlickeiser 2005).

Instabilities correspond to complex frequencies $\omega(\boldsymbol{k})$ with a positive imaginary part, due to the $\mathrm{e}^{-\mathrm{i}\omega t}$ Fourier approach for the electromagnetic fields (in the case of aperiodic modes, $\omega = \mathrm{i}\Gamma$ with $\Gamma > 0$ is purely imaginary). On many occasions, excellent agreement between the solutions of dispersion relations such as Eq. (1) and numerical particle-in-cell simulations (Büchner et al. 2010; Bunemann 1993) has been demonstrated (e. g., Tautz & Sakai 2007).

For the solar wind, it has even been argued that aperiodic fluctuations contribute a significant fraction of the turbulence (Stockem et al. 2006), which is backed by observations that the two-dimensional component (as discussed below) appears to be time-independent. If we now consider fully evolved turbulence – in the sense of being quasi-stationary and homogeneous – it is sufficient for many purposes to limit the investigation to some general properties of the turbulence (Batchelor 1982), which are (i) the (Fourier) power spectrum, often denoted as $G(k)$; (ii) the dynamic behavior (see Shalchi 2009); and (iii) the turbulence geometry. Common models are isotropic turbulence or, as motivated both by theoretical arguments and observational constraints, a superposition of two orthogonally components – the so-called composite model, which superposes the two orthogonal directions parallel and perpendicular to the preferred axis. A recent careful investigation shows that more realistic, fully three-dimensional models such as the Maltese cross (Matthaeus et al. 1990; Rausch & Tautz 2013; Tautz 2012a) are required because the resulting diffusion parameters can significantly differ from that obtained with the composite model.

An analytical treatment requires the use of two-point, two-time correlation functions as $\langle \delta\boldsymbol{B}(\boldsymbol{r}_1, t_1)\, \delta\boldsymbol{B}(\boldsymbol{r}_2, t_2)\rangle$, which, in Fourier space, leads to the correlation tensor

$$\mathsf{P}_{\ell m}(\boldsymbol{k})\, \Gamma(\boldsymbol{k}, t) = \langle \delta\boldsymbol{B}_\ell(\boldsymbol{k}, t)\, \delta\boldsymbol{B}_m^\star(\boldsymbol{k}, t)\rangle\,, \tag{2}$$

with $\ell, m \in \{x, y, z\}$. The advantage is that, as can be derived using the general theory of correlation functions (see references in Tautz & Shalchi 2010b), the correlation tensor leads back to the power spectrum via

$$\mathsf{P}_{\ell m}(\boldsymbol{k}) = \frac{G(\boldsymbol{k})}{8\pi k^2} \left(\delta_{\ell m} - \frac{k_\ell k_m}{k^2} + \ldots\right), \tag{3}$$

where additional terms are neglected for simplicity. A direct relation between the fluctuating magnetic field and the power spectrum (and the turbulence geometry) can therefore be obtained.

Measurements in the solar wind taken for example by the Helios spacecraft (see Bruno & Carbone 2005, and references therein) indicate that there is a pronounced frequency/wavenumber regime with a Kolmogorov-type (1991) turbulence cascade, i. e., with $G(k) \propto k^{-5/3}$. Unfortunately, the shape of both the energy range ($k \lesssim 0.03\,\mathrm{AU}^{-1}$) and the dissipation range ($k \gtrsim 3\times 10^6\,\mathrm{AU}^{-1}$) is considerably less clear. To account for a variable of course, the above considerations barely scratch the surface of turbulence and by no means represent the only accepted model. Another well-known example is the Iroshnikov-Kraichnan $2/3$ spectrum (see Kraichnan 1973; Ng et al. 2010, and references therein).

3 Test-particle transport

If the transport of a few additional, high-energetic particles should be traced inside an otherwise smooth turbulent magnetized plasma, we first need to limit ourselves to a test-particle approach in which the scattered particles do not back-react on the turbulence. In addition, one always has to consider an *ensemble* of such particles as the trajectory of a single particle is not meaningful due to the randomness inherent in any turbulent behavior.

Therefore, a statistical approach is required, which can be met by formulating a transport equation (Earl 1976; Luhmann 1976; Parker 1965; Roelof 1969)

$$\frac{\partial f}{\partial t} = \nabla \cdot (\kappa_{\ell m} \cdot \nabla f) + \nabla_{\boldsymbol{p}} \left(p^2 D_p \nabla_{\boldsymbol{p}} \frac{f}{p^2} \right) + \dots, \tag{4}$$

where $\kappa_{\ell m}$ denotes the spatial diffusion tensor and where D_p is the momentum diffusion coefficient. From the structure of the equation is can be seen that the analysis is based on the diffusion approach but, due to the preferred direction imposed by the mean magnetic field, the spatial diffusion coefficient is no longer a scalar. In addition, fluctuating electric fields can cause stochastic variations in the the particle energy, which typically leads to acceleration (Tautz 2010b).

For a general background magnetic field, $\boldsymbol{B}$, the spatial diffusion tensor takes the form (Giacalone et al. 1999):

$$\kappa_{ij} = \kappa_\perp \delta_{ij} - \left(\kappa_\perp - \kappa_\parallel \right) \frac{B_i B_j}{B^2} + \kappa_{\mathrm{A}} \epsilon_{ijk} \frac{B_k}{|B|}, \tag{5}$$

with $\kappa_\parallel$ and $\kappa_\perp$ describing diffusion along and across the magnetic field, respectively, and κ_{A} covering drift effects due to magnetic field curvature (Tautz & Lerche 2011; Tautz & Shalchi 2012). In order to solve the transport equation, the diffusion coefficients have to be known in advance. Unfortunately, however, it turns out that they depend sensitively on the details of the electromagnetic turbulence, to a degree that even the "diffusivity" of the system is not always fulfilled (Tautz & Shalchi 2010a).

3.1 Test-particle simulations

One possible approach to obtain the diffusion coefficients simply from the trajectories of charged particles being scattered by turbulent fields is found in test-particle (or *full-orbit*) simulations, where the equation of motion of a large number of particles is integrated (Giacalone & Jokipii 1999; Tautz 2010a; and references in Tautz & Dosch 2013). In what follows, use is made of the fact that the particles' mean-square displacement is connected to the diffusion coefficients via

$$\kappa_\parallel(t) = \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} \left\langle (\Delta z(t))^2 \right\rangle \approx \frac{1}{2t} \left\langle (\Delta z(t))^2 \right\rangle, \tag{6}$$

and also to the mean-free path via $\kappa = v\lambda/3$. It is therefore possible to average over a sufficiently large number of particle displacements and so obtain not only diffusion coefficients but also drift coefficients and Fokker-Planck coefficients, e. g., for pitch-angle scattering (see below).

The word "test" in test-particle simulation indicates that the particles do not back-react on the fields. Accordingly, the turbulent fields have to be specified externally, which is usually done by the superposition of plane waves as (Batchelor 1982; Tautz & Dosch 2013)

$$\begin{aligned} \delta \boldsymbol{B}(\boldsymbol{r}, t) \propto & \sum_{n=1}^{N} \hat{\boldsymbol{e}'}_{\perp,n} \sqrt{G(k_n)\, \Delta k_n} \\ & \times \cos\left[\mathrm{i}\left(k_n z' - \omega(k_n) t + \beta_n\right)\right], \end{aligned} \tag{7}$$

where β is the random phase of each wave and $\hat{\boldsymbol{e}'}_\perp$ and $\hat{\boldsymbol{e}}'_z$ are the directions of the wave polarization and propagation, respectively, with $\hat{\boldsymbol{e}'}_{\perp,n} \perp \hat{\boldsymbol{e}}'_z\ \forall n$ so that the field is divergence-free. The wavenumbers k_n are logarithmically spaced so that $k_n\, \Delta k_n$ is constant, thus covering a large range of spatial scales. Dynamical effects – typically in the form of Alfvén or magnetosonic waves (Tautz 2010b; Tautz et al. 2006) – are included via a dispersion relation $\omega(k)$, which, for magnetostatic turbulence, can be set to zero.

Before we come to the more intricate details of the particle behavior, there is an interesting question connected to Eq. (7): How many wave modes need to be superposed in order to obtain a sufficiently "turbulent" behavior? To answer this question, the onset of stochasticity has been investigated using a quasi-Lyapunov approach (Tautz & Dosch 2013), which showed that – depending on the level of structure in the turbulence – a minimum of eight wavenumbers already results in a behavior that is qualitatively identical to that obtained for 10^2–10^3 wave modes.

3.2 Test-particle theory

For analytical investigations, it has proven useful to introduce the pitch-angle cosine, $\mu = \cos\angle(\boldsymbol{v}, \boldsymbol{B}_0)$, as a basic coordinate because it can be linked to scattering along the mean magnetic field. Starting from the Vlasov equation, the test-particle approach results in a Fokker-Planck equation, where the Fokker-Planck coefficients are defined through

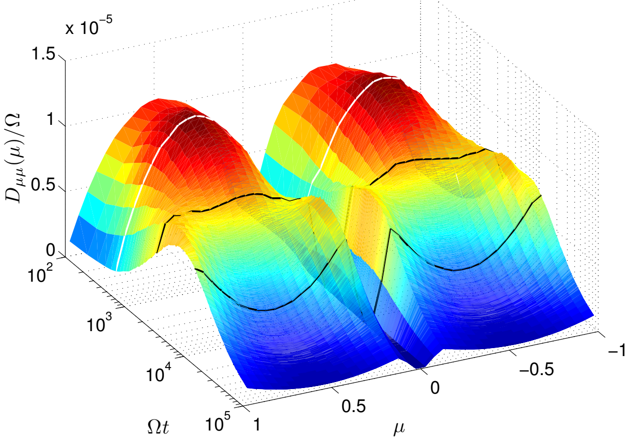

Fig. 1 Time-dependent Fokker-Planck coefficient of pitch-angle scattering, $D_{\mu\mu}(\mu,t)$ for particles width rigidity $R=10^{-2}$ in moderate turbulence strength, $\delta B/B_0=10^{-1.5}$. The white solid line shows the well-known "double-hump" structure, whereas the black solid lines illustrate the behavior of $D_{\mu\mu}(\mu)$ at later times (cf. Tautz et al. 2013).

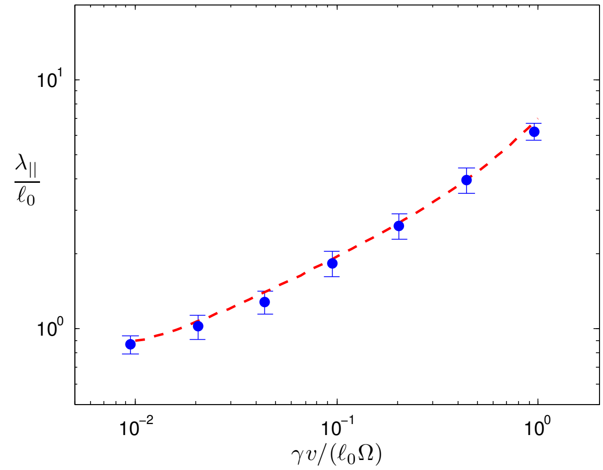

Fig. 2 The parallel scattering mean-free path as a function of the particle speed. Shown are the simulation results from the PADIAN code (blue dots with error bars) in comparison with analytical results (red dashed line) that have been derived using the second-order quasi-linear theory (Tautz et al. 2008b).

the time-integral of the two-time correlation function of all parameter combinations. For the pitch-angle Fokker-Planck coefficient, one therefore has

$$D_{\mu\mu}=\int_0^t \mathrm{d}t' \,\left\langle \dot\mu(t')\,\dot\mu(0)\right\rangle, \tag{8}$$

where the time derivative of the pitch angle can be expressed through the equation of motion as

$$\dot\mu \propto \frac{1}{v}\left(v_x\,\frac{\delta B_y(\boldsymbol{r},t)}{B_0}-v_y\,\frac{\delta B_x(\boldsymbol{r},t)}{B_0}\right). \tag{9}$$

Inserting $\dot\mu$ in the Fokker-Planck coefficient in Eq. (8) leads back to the correlation function, which can be expressed through the turbulence power spectrum. The problem, however, is that the position and velocity vectors in Eq. (9) are, in general, unknown. This is why perturbation theories such as the quasi-linear theory (Jokipii 1966) have to be used, which make drastic assumptions such as an unperturbed spiral orbit for the particle motion. Non-linear extensions, on the other hand, often lack the analytical tractability and thus require additional simplifications (see Shalchi 2009, for an overview).

Although it is not completely straightforward, it is possible to extract the Fokker-Planck coefficient for pitch-angle scattering from the trajectories of simulated test particles (Qin & Shalchi 2009; Tautz et al. 2013). A peculiarity of the pitch-angle diffusion parameter is the fact that it depends on the pitch-angle itself. Therefore, clever sorting (and better statistics) are required during the numerical evaluation. The comparison with analytical theories shows that, in principle, agreement can be achieved, but only if the correct time point is chosen (see Fig. 1). For long times, in contrast, no agreement at all is found.

From the Fokker-Planck coefficient, the mean-free path along the magnetic field can be evaluated as (e. g., Earl 1974; Hasselmann & Wibberenz 1968)

$$\lambda_\parallel=\frac{3v}{8}=\int_{-1}^{1}\mathrm{d}\mu\,\frac{\left(1-\mu^2\right)^2}{D_{\mu\mu}(\mu)}, \tag{10}$$

which formula is most often used in analytical derivations. Numerically, the detour over pitch-angle scattering can be skipped and the considerably simpler evaluation according to the parallel mean-square displacement can be used as given in Eq. (6). In that case, excellent agreement (see Fig. 2) can be found between analytical theory (Tautz & Lerche 2010; Tautz et al. 2008b) and numerical simulation (Tautz 2012b) but only if a non-linear theory (Shalchi 2005) is used. Quasi-linear theory, in contrast, leaves us with an infinitely large mean-free path, which again underlines that, in some cases, its results are not only inaccurate but instead may be plainly invalid.

The fact that, even though the comparison in the case of the mean-free path yields a surprisingly convincing result, the Fokker-Planck coefficient shows a temporal behavior that is not found in the analytical theory, has recently been analyzed in more detail (Tautz 2013). It was found that the usual analog between spatial and pitch-angle diffusion is not permitted in general for two reasons: (i) the mean-square displacement cannot grow indefinitely as required by

$$D_{\mu\mu}=\frac{1}{2}\,\frac{\mathrm{d}}{\mathrm{d}t}\left\langle(\Delta\mu)^2\right\rangle\approx\frac{1}{2t}\left\langle(\Delta\mu)^2\right\rangle\overset{!}{=}\text{const}; \tag{11}$$

(ii) the assumption of homogeneity in time, which is central to the derivation of the above equation, cannot be fulfilled by a diffusion process in a space as restricted as pitch-angle space due to $-1\leqslant\mu\leqslant 1$. Accordingly, a more complicated description has to be used as shown by Tautz (2013).

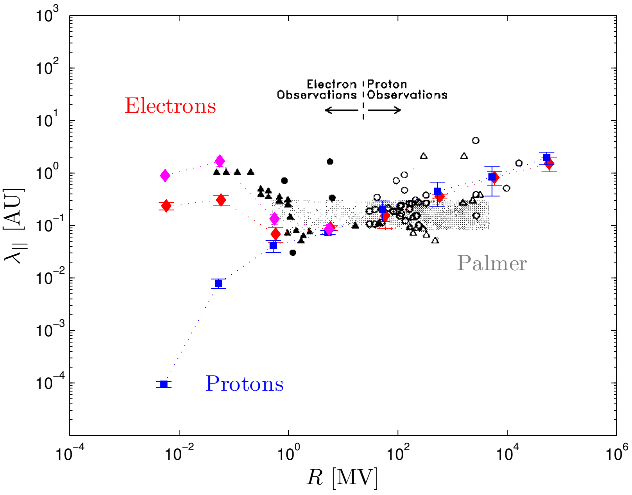

Fig. 3 The parallel mean-free path as a function of the particle rigidity for electrons (red and magenta diamonds) and for protons (blue dots) from the numerical simulations using the PADIAN code. The comparison with the observational data (Palmer consensus range, see Bieber et al. 1994; Palmer 1982) shows excellent agreement (Tautz & Shalchi 2013). Data points are obtained from Fig. 1 of Bieber et al. (1994).

4 Solar system

As usual, reality is more complicated than the idealized scenarios presented in the previous sections. If, as a nearby example, we consider the solar system, we have to include various additional phenomena such as:

- the curvature of the mean magnetic field line as given by the Parker (1958, see also Burger et al. 2008 for a more sophisticated modeling) spiral;
- the effects of a dynamical turbulence on short time scales as caused predominantly by magnetohydrodynamic waves such as Alfvén and magnetosonic waves;
- the random walk of magnetic field lines as these determine the behavior of charged particles (Tautz et al. 2008a) and might be accessible to the inclusion of magnetic reconnection effects;
- intermediate and long-term dynamics as given by intermittent turbulence and the solar cycle, respectively;

All these effects are known to have a severe impact on the particle behavior and so have to be incorporated; even worse, there is no ordering that would allow for a prediction as to which of the effects might be neglected. A realistic scenario (Tautz & Shalchi 2013) should therefore combine certain key aspects into an anisotropic turbulence model, among which are: (i) the dynamic behavior of shear Alfvén waves in a parallel slab component (Belcher & Davis 1971); (ii) the approximately time-independent behavior of a two-dimensional component that is argued to be formed by the aperiodic plasma modes generated by Weibel-type instabilities (Stockem et al. 2006); (iii) collisionless dissipation, which which sets apart electrons from heavier particles such as protons. Additional parameters to be taken from observations are the turbulence bend-over scale, which forms the onset of the Kolmogorov turbulence cascade, the background magnetic field strength, and the relative turbulence strength, which, strictly speaking, is a function of the radial distance from the Sun (Tautz et al. 2011).

From cosmic-ray observations in the solar system, scattering mean-free paths can be obtained by fitting the observed time profiles to diffusion models. Palmer (1982) concluded that, for rigidities between 0.5 and 5000 MV, the parallel mean-free path is $0.08\,\mathrm{AU} \leqslant \lambda_{\parallel} \leqslant 0.3\,\mathrm{AU}$, which has been named the "Palmer consensus range" (Bieber et al. 1994). In Fig. 3, the original measurements are shown in addition to the Palmer consensus range. The numerical results (Tautz & Shalchi 2013) obtained with the aforementioned model compare well with the experimental data, even beyond the range of validity of Palmer consensus, where the mean-free paths are substantially larger than those summarized in the Palmer consensus range.

5 Summary and conclusion

Whenever the need arises to describe the transport of charged particles through turbulent electromagnetic fields, one is confronted with a three-dimensional, anisotropic diffusion-like process. On the one hand, some researchers are interested in fundamental problems related both to the origin of the highest-energetic particles found in nature and to the dynamics of tenuous magnetized plasmas and wave-particle interactions. One the other, there are many practical issues involved, among which the prediction of space weather is of particular importance. Furthermore, the influence of cosmic rays on Earth's climate (Scherer et al. 2006), on the survivability of organic molecules on Mars, and, via the driving of Galactic winds, on the evolution of the Galaxy as a whole, are actively discussed.

In this article, the basic concepts have been summarized that are required for the description of wave-particle interaction, where (i) plasma waves are generated via (linearized) instabilities so that free bulk energy is converted into small-scale turbulent electromagnetic fields; and (ii) the random motion of charged particles in such electromagnetic fields, which, due to the turbulent nature of the fields, is meaningful only via ensemble averages.

In future work, we plan to include the effects of additional time scales, which can be caused by (Landau) damping of the electromagnetic waves constituting the turbulence or by intermittency effects, i. e., spatial regions where the degree of turbulence is significantly reduced. The latter can be described in terms of a Lévy random walk, thereby requiring a fractal Fokker-Planck equation. In addition, other forms of turbulence such as the direct superposition of waves as given through (measured) frequency spectra are possible as well as the inclusion of magnetic reconnection, by which additional acceleration effects will be introduced.

Furthermore, by giving up spatial homogeneity it is possible to include (collisionless) shock waves. This enables one to place almost all the aforementioned effects in the context of Fermi acceleration and so investigate the origin not only of cosmic rays but also of particles that are accelerated in the solar system by transient phenomena such as shocks driven by coronal mass ejections (Zank et al. 2006; Chpt. 8.2 of Shalchi 2009, and references therein). Finally, the evaluation of time profiles directly within the simulations will allow for a more direct comparison to spacecraft data not only for solar or Galactic cosmic rays, but also for particles with lower energies as found, for example, in the vicinity of the Jovian magnetosphere.

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