Models of Experimental Fluid Dynamos

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Abstract

Several experiments are presently in preparation which intend to reproduce a homogeneous dynamo on a laboratory scale. This paper reviews the theoretical models on which some of these experiments are based.

1 Introduction

The dynamo effect is commonly invoked in astrophysics and the planetary sciences in order to explain the generation of large scale magnetic fields. Despite the apparent omnipresence of this effect in the universe, no experimental demonstration of a homogeneous dynamo reproducing at least qualitatively essential features of natural dynamos has been successful up to now. It is indeed a major technological challenge to build a homogeneous dynamo on a laboratory scale. A growing number of groups is presently engaged in setting up dynamo experiments. The sudden surge in experimental activities has been helped by the outphasing of fast nuclear breeder technology in most industrialized countries so that the know how and the hardware necessary for handling large volumes of liquid sodium become available for other purposes. Technical aspects of dynamo experiments are covered in a recent review (Tilgner 2000). The present contribution focuses on the theoretical models underlying those experiments whose actual realizations are closest to completion at the time of writing.

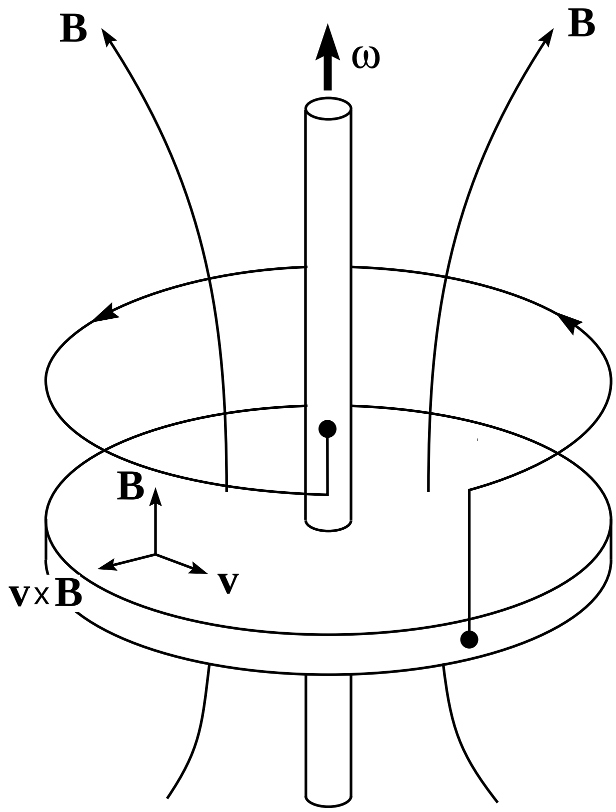


Figure 1: The disc dynamo

2 The disc dynamo and general concepts

The simplest dynamo (i. e. the simplest engine converting mechanical into magnetic energy) is the disc dynamo depicted in figure 1. The entire structure is made of conducting material. A disc rotates at angular velocity $\boldsymbol{\omega}$ about an axis, and a wire electrically connected by brushes to the disc and the axis allows current to flow in a closed loop from the rim of the disc to the axis and back to the disc. If an initial seed field of roughly dipolar structure permeates the rotating disc as indicated in the figure, an electromotive force $\boldsymbol{v} \times \boldsymbol{B}$ (where $\boldsymbol{v}$ is the local velocity of the disc) is directed everywhere towards the rim of the disc and drives a current through the wire. The wire is shaped such that the current flowing in it induces a magnetic field which superposes constructively with the initial seed field. In this qualitative picture, the strengthened magnetic field drives a larger current which in turn leads to a yet stronger magnetic field, etc. The magnetic field grows without end if the disc rotates at a prescribed angular velocity. In a model closer to reality where the axis might be connected to a motor exerting a finite torque, the rotation rate of the axis actually decreases when the magnetic field grows because the electromagnetic force $\boldsymbol{j} \times \boldsymbol{B}$ (where $\boldsymbol{j}$ is the local current density in the disc) opposes the rotation of the disc.

The disc dynamo exhibits the essential features common to all dynamos. The dynamo effect needs a seed field to start. Zero magnetic field is always a solution, albeit possibly an unstable one. Whether it is unstable or not depends on the structure and the vigor of the motion in the conductor: if the rotation rate of the disc is too slow, ohmic dissipation depletes the magnetic field too quickly and overwhelms the generation mechanism described above. In addition, one obtains another consistent picture after inverting the arrows for the magnetic field and the current in figure 1. Indeed, if a motion supports a magnetic field $\boldsymbol{B}(\boldsymbol{r})$, it is also able to generate $-\boldsymbol{B}(\boldsymbol{r})$. The initial conditions decide the polarity. Finally, if the motion of the conductor is prescribed (the so called "kinematic dynamo" problem), the magnetic field decays or grows indefinitely. If on the contrary a driving force is prescribed, a dynamo field saturates at a level such that a balance is struck between the Lorentz force, the driving force and friction.

The disc dynamo relies for its operation on the shape of the wire. If it were plunged into a bath of liquid metal, disc and axis would be short circuited and no magnetic field could be generated. Since planetary cores and stellar interiors are essentially homogeneous masses of fluid conductor, they appear at first sight as very unlikely places for a dynamo to operate. A "homogeneous dynamo" is a fluid flow which generates a magnetic field despite the electric current being free to flow within a simply connected (for example spherical) volume. The current experimental aim is to construct such homogeneous dynamos. Inhomogeneous dynamos exist since long and are found in any car or on bicycles.

Dynamos operating in liquid metals moving at velocities $\boldsymbol{v}(\boldsymbol{r})$ small compared with the speed of light are well described by the induction equation of magnetohydrodynamics

$$\frac{\partial}{\partial t}\boldsymbol{B} + \nabla \times (\boldsymbol{B} \times \boldsymbol{v}) = \lambda \nabla^2 \boldsymbol{B} \quad , \quad \nabla \cdot \boldsymbol{B} = 0 \tag{1}$$

where λ is a material constant (the magnetic diffusivity) assumed constant in (1) throughout the conductor. (1) describes the kinematic dynamo problem. Additional equations (e. g. the Navier-Stokes equation) are necessary to determine how $\boldsymbol{v}$ reacts to a growing magnetic field. Since the present challenge is to generate magnetic fields at all, we will restrict our attention to (1). The following sections describe solutions of (1) which have motivated experiments under construction.

3 The Riga experiment

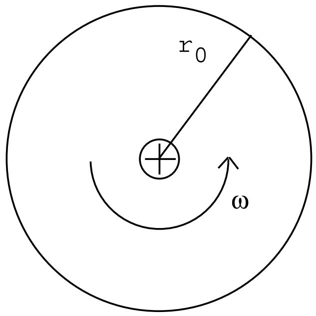
The first attempt at building a homogeneous dynamo with liquid sodium has been undertaken by the Riga group (Gailitis et al. 1987, Gailitis 1990). The first experiment has failed to produce a dynamo, but an improved apparatus based on the same theoretical model is being set up. The experiment consists of three coaxial pipes. In the inner pipe sodium is flowing along helical streamlines (at least on average, the flow being turbulent). In the middle pipe, the fluid returns from the exit of the inner pipe to its entrance. The outer pipe contains sodium at rest. This configuration is inspired by a model introduced by Ponomarenko (1973) who studied a fluid moving with velocity $\boldsymbol{v} = \omega r \hat{\boldsymbol{\varphi}} + v_z \hat{\boldsymbol{z}}$ in a cylindrical region $r \leq r_0$ (in cylindrical polar coordinates r, z, φ) and $\boldsymbol{v} = 0$ for $r > r_0$ (fig. 2 a). The differences between the model and the experiment are that the pipes have finite length, the conductor at rest in the experiment does not extend to infinity radially, and there is no return flow in the model. Closer approximations to the experiment than shown in fig. 2 a have of course been investigated, but the basic field growth mechanism is already contained in the Ponomarenko model.

The induction equation (1) with the velocity field under consideration can be solved by the ansatz $\boldsymbol{B}(\boldsymbol{r}, t) = \boldsymbol{b}(r) e^{\sigma t} e^{ikz} e^{im\varphi}$. A growth rate σ with positive real part indicates a working dynamo. The magnetic fields excited most easily have an azimuthal wavenumber m of 1. Since the z-dependence is assumed periodic with wavenumber k, there only remains the radial dependence to be determined. In each of the regions $r \leq r_0$ and $r > r_0$, the equations for the components of $\boldsymbol{b}(r)$ are solved by linear combinations of Bessel functions I and K. The partial differential equation (1) is then reduced to an algebraic equation for a few coefficients. Only at this point is it necessary to resort to numerical computation in order to find combinations of ω, v_z, r_0, λ and k such that σ has a positive real part. Such combinations do exist.

It being established that the Ponomarenko flow is a dynamo for a suitable choice of parameters, one still would like to have a qualitative picture of how it works similar to that presented for the disc dynamo above. By a suitable choice of frame of reference, the velocity field in either of the regions $r \leq r_0$ or $r > r_0$ may be transformed to zero. It follows that the magnetic field in both regions is just diffusing ((1) with $\boldsymbol{v} = 0$ is a diffusion equation for $\boldsymbol{B}$). The entire generation mechanism must therefore be concentrated at the interface between the two regions. It is now useful to invoke Alfvén's "frozen flux theorem" which states that in a perfect conductor, magnetic field lines are carried along by the fluid. In a real fluid, a magnetic field line which connects two fluid particles will not do so forever but the deformation of the field lines due to fluid motion is still reminiscent of the frozen flux dynamics. Coming back to the Ponomarenko dynamo, assume that the radial component B_r is different from zero at the interface $r = r_0$ so that magnetic field lines are crossing from one region to the other. Because of the strong shear, the field lines get aligned with the cylindrical surface $r = r_0$. Elongation of field lines leads to amplification of the φ and z components of $\boldsymbol{B}$. The shear may therefore lead to strong B_φ and B_z from a weak B_r, but how is B_r replenished? Interestingly enough, this happens by pure diffusion. The radial component of the induction equation multiplied by B_r and integrated over the periodicity volume V limited by $0 \le z \le 2\pi/k$, $0 \le \varphi \le 2\pi$, $0 \le r \le \infty$ leads to

$$\frac{d}{dt}\int \frac{1}{2} B_r^2 \, dV = -\lambda \int (\nabla B_r)^2 \, dV - \lambda \int \frac{1}{r^2} B_r^2 \, dV - \lambda \int \frac{2}{r^2} B_r \partial_\varphi B_\varphi \, dV .$$

The first two terms on the right hand side are negative. The third term also arises from the diffusion term but it may be positive. Therefore, dynamo fields in the Ponomarenko model have a radial component of $\boldsymbol{B}$ fed through diffusion by the φ-component which it is amplified through shearing motion at $r = r_0$. The diffusive coupling between B_r and B_φ is due to the curvature of the cylindrical surface $r = r_0$. This picture is supported by the more general analysis of Ruzmaikin and Sokoloff (1980) and the asymptotics by Roberts (1987) and Gilbert (1988).



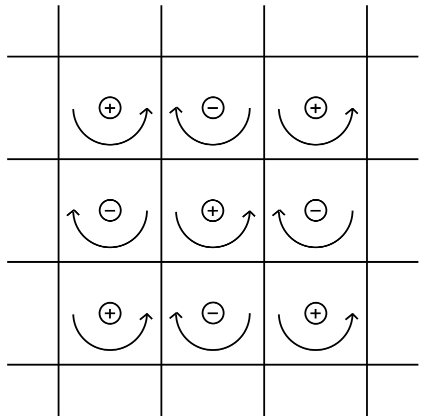


Figure 2 a: The Ponomarenko dynamo. Within a cylinder of radius r_0, the conductor is rotating like a solid body at angular velocity ω and translating along the axis as indicated by the $+$-sign. The conductor is at rest for $r > r_0$. b: Nine cells of an infinite periodic two dimensional dynamo introduced by G. O. Roberts. Just as in a), the signs indicate the direction of the translation (the out of plane velocity component in the figure) and the arrows show the sense of the swirling motion, i. e. the in plane component.

4 The Karlsruhe experiment

The Karlsruhe experiment emphasizes the concept of scale separation: motion on a small spatial scale occurs within a much larger volume of fluid. Motion on many different scales is found in turbulent flows, too. Convection in rapidly rotating fluids (e. g. planetary interiors) also occurs in columns aligned with the rotation axis, where the radius of each column is much less than the radius of the fluid boundaries and where the motion within columns is roughly helical. The flow realized in the Karlsruhe experiment reproduces some of these features.

The Karlsruhe experiment represents a finite excerpt of a model introduced by G. O. Roberts (1972) (fig. 2 b), who considered an array of right handed helical swirls periodic in the x, y-plane. The induction equation for this velocity field needs to be solved numerically, but there is a limit for which a useful analytic approximation is known. Numerical computations show that the periodic flow in fig. 2b preferentially generates a magnetic field which has the same periodicity as the velocity field (Tilgner and Busse 1995). Such a magnetic field includes a component which is independent of x and y and which is referred to as the "large scale" magnetic field. In a realization of finite extent of fig. 2 b, this component would also vary in x and y due to boundary conditions, but more slowly than the rest of the field which varies on the length scale of the periodicity length of the flow. Hence the designation as "large scale field" and its distinction from "small scale" or "fluctuating" components. If the magnetic Reynolds number on the scale of a periodicity cell is small, the large scale field is much larger than the fluctuating part. The limit of small magnetic Reynolds number can be exploited using different mathematical formalisms, either in the Fourier domain (G. O. Roberts 1972, Gubbins 1974), by "smoothing" (Ghil and Childress 1987) or by averaging (Busse 1992, Busse et al. 1998, Rädler et al. 1998). The result is that if one decomposes the magnetic field $\boldsymbol{B}$ into a large scale field $\bar{\boldsymbol{B}}$ and a small scale field $\boldsymbol{b}$ as $\boldsymbol{B} = \bar{\boldsymbol{B}} + \boldsymbol{b}$, one obtains (at lowest order) an equation for $\bar{\boldsymbol{B}}$ alone:

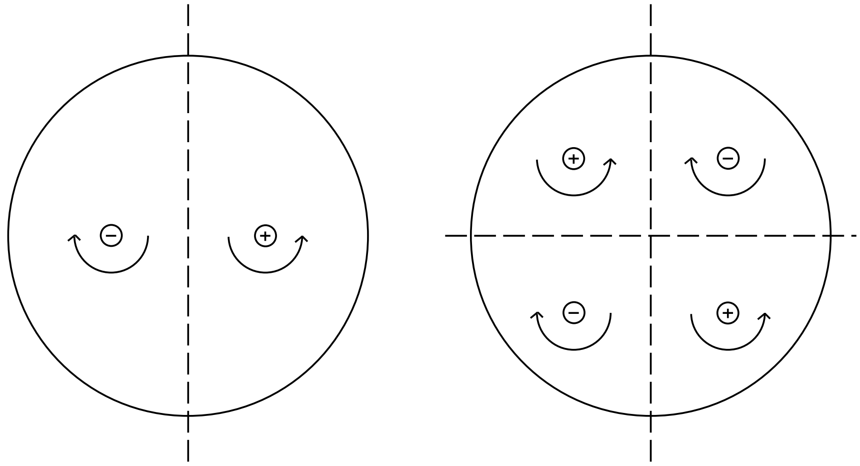
$$\partial_t \bar{\boldsymbol{B}} + \nabla \times \bar{\boldsymbol{E}} = \lambda \nabla^2 \bar{\boldsymbol{B}}$$

where the large scale electromotive force $\bar{\boldsymbol{E}}$ is given by

$$\bar{\boldsymbol{E}} = \alpha \bar{\boldsymbol{B}},$$

α being a tensor. The generation of a large scale $\bar{\boldsymbol{E}}$ by a large scale $\bar{\boldsymbol{B}}$ is known as the "α-effect". The expression for α subsumes the dynamics on the small scales. Qualitatively, the dynamo works as follows: Starting from a suitable large scale seed field, a current is generated through the α-effect which itself is source for a large scale field which superposes constructively with the initial seed field, thus completing again a dynamo cycle akin to those identified for the disc and Ponomarenko dynamos. For a detailed presentation, see Busse et al. (1998).

In the Karlsruhe experiment, the Reynolds number on the scale of a single eddy at the onset of dynamo action is expected to be in the range 8–10, which is not small. Quantitative modeling thus requires either retaining many terms in an expansion in the magnetic Reynolds number (Rädler et al. 1998) or direct simulation of the full induction equation (Tilgner 1997).



a b

Figure 3: Two motions studied numerically by Dudley and James. The flows are axisymmetric about the vertical axis indicated by a dashed line. In order to emphasize the relationship with the other dynamos, meridional (in plane) and azimuthal (out of plane) components of the velocity field are indicated in the same way as in figure 2.

5 Models of Dudley and James

At least three groups are in the process of building experiments which reproduce flows studied numerically by Dudley and James (1989) (for a review of the experiments see Tilgner 2000). These are axisymmetric flows in a sphere consisting of one or two eddies (fig. 3 a, b). There is as yet no simple understanding of the dynamo mechanism in these flows and their investigation has been a purely numerical matter so far. However, they can probably be linked to the dynamos of the previous sections. The circulation of fig. 3 a for instance is a Ponomaranko dynamo whose cylindrical axis has been wrapped azimuthally around the vertical symmetry axis of figure 3. Alternatively, this flow is topologically identical to the one in the Riga experiment with the pipes aligned vertically. One can also imagine a continuation of the sequence of figures 3a and b leading to an axisymmetric flow consisting of many cells. Such a flow is not practical for an experiment, but it becomes again amenable to mean field analysis (Gubbins 1973). Analogies between the magnetic fields generated by the Dudley and James models and other dynamos have apparently not been explored yet.

6 Conclusion

The experiments due to run in the near future are based on a variety of models and on different excitation mechanisms for the magnetic field. The non linear effects leading to the saturation of magnetic field growth are expected to be equally varied. Issues of field saturation are best settled by the experiments themselves. At the end of the year 1999 both the Riga and the Karlsruhe groups claimed to have observed a self-exited magnetic dynamo field in their respective apparatus.

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